MATH 450
Fall 2018
Practice problems
10/23/18
Time Limit: 40 Minutes

Name (Print): ____________________

This exam contains 4 pages (including this cover page) and 3 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books/notes on this exam. You may use calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Points	Score
1	30	
2	30	
3	40	
Total:	100	

Do not write in the table to the right.

1. For each part, write a brief explanation in the space provided. For true/false questions, also circle true/false.

   (a) (15 points) Assume that the distribution of X is $\mathcal{N}(46.58, 40.96)$. Let $\bar{X}$ be the sample mean of a random sample of $n = 16$ observations of X.

   (i) Give the values of $E[\bar{X}]$ and $Var[\bar{X}]$

   (ii) Compute $P[44.42 \leq \bar{X} \leq 48.98]$

   (b) (15 points) From the same distribution P, two independent random samples of size n

$$x_1, x_2, \ldots, x_n \quad \text{and} \quad x'_1, x'_2, \ldots, x'_n$$

   are collected. For each of the two data sets, a 95% confidence interval for the mean μ is constructed. What is the probability that both interval contains the true mean μ?

2. (30 points) Recalling that the probability density function of a $\mathcal{N}(1, \sigma^2)$ random variable is

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-1)^2}{2\sigma^2}},$$

derive the maximum-likelihood estimator for parameter σ based on a $\mathcal{N}(1, \sigma^2)$ sample of size n

$$x_1, x_2, \ldots, x_n.$$

3. A random sample of size $n = 10$ is collected from a normal distribution with population mean μ. The collected data are:

$$25.2,\ 21.3,\ 22.8,\ 17.0,\ 29.8,\ 21.0,\ 25.5,\ 16.0,\ 20.9,\ 19.5$$

   (a) (20 points) Estimate the population mean μ in a way that conveys information about precision and reliability

   (b) (20 points) Suppose that another single observation, X_{11}, is to be selected from the same distribution, predict X_{11} in a way that conveys information about precision and reliability.