MATH 450
Fall 2018
Review problems
12/03/18
Time Limit: 60 Minutes

Name (Print): ____________________

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books/notes on this exam. You may use calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	60	
3	20	
4	25	
5	15	
Total:	140	

1. (20 points) A gas station sells three grades of gasoline: regular unleaded, extra unleaded, and super unleaded. These are priced at 2.20, 2.35, and 2.50 per gallon, respectively. Let X_1, X_2, and X_3 denote the amounts of these grades purchased (gallons) on a particular day. Suppose the X_i's are independent normal random variables with $\mu_1 = 1000$, $\mu_2 = 500$, $\mu_3 = 300$, $\sigma_1 = 100$, $\sigma_2 = 80$, $\sigma_3 = 50$. What is the probability that the revenue

$$Y = 2.2X_1 + 2.35X_2 + 2.5X_3$$

exceeds 4500?

We first compute the expected value and variance of Y.

$$\mu_Y = 2.2E[X_1] + 2.35E[X_2] + 2.5E[X_3] = 4125,$$
$$V[Y] = 2.2^2V[X_1] + 2.35^2V[X_2] + 2.5^2V[X_3] = 99369$$

and $\sigma_Y = 315.23$.

Thus,

$$\begin{aligned} P[Y > 4500] &= P\left[\frac{Y - \mu_Y}{\sigma_Y} > \frac{4500 - 4125}{315.23}\right] \\ &= P\left[Z > 1.19\right] \\ &= 1 - P\left[Z \leq 1.19\right] \\ &= 0.1170 \end{aligned}$$

2. (20 points) Suppose that against a certain opponent, the number of points a basketball team scores is normally distributed with unknown mean μ and unknown variance σ^2. Suppose that over the course of the last 10 games, the team scored the following points:

$$59, \quad 62, \quad 59, \quad 74, \quad 70, \quad 61, \quad 62, \quad 66, \quad 62, \quad 75$$

(a) (20 points) Compute a 95% confidence interval for μ.

We have $\bar{x} = 65, s = 5.98$ and $t_{0.025,9} = 2.262$

$$\left(\bar{x} - t_{0.025,9}\frac{s}{\sqrt{n}}, \bar{x} + t_{0.025,9}\frac{s}{\sqrt{n}}\right)$$

(b) (20 points) Now suppose that you learn that $\sigma^2 = 25$. Compute a 95% confidence interval for μ. How does this compare to the interval in (a)?

We have $\bar{x} = 65, \sigma = 5$ and $z_{0.025} = 1.96$

$$\left(\bar{x} - z_{0.025}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{0.025}\frac{\sigma}{\sqrt{n}}\right)$$

3. (20 points) In an experiment designed to measure the time necessary for an inspectors eyes to become used to the reduced amount of light necessary for penetrant inspection, the sample average time for $n = 9$ inspectors was 6.32s and the sample standard deviation was 1.65s.

   It has previously been assumed that the average adaptation time was 7s. Assuming adaptation time to be normally distributed, does the data contradict prior belief? Use the t test with $\alpha = 0.1$.

   Denote the mean adaptation time by μ. We are testing

$$H_0 : \mu = 7$$
$$H_a : \mu \neq 7$$

   Since σ is unknown and sample size is small, we use the t-test. For $\alpha = 0.1$ and $df = 9 - 1 = 8$, we have $t_{0.05,8} = 1.86$ and the rejection region is:

$$t \geq 1.86 \quad \text{or} \quad t \leq -1.86$$

   On the other hand

$$t = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{6.32 - 7}{1.65/\sqrt{9}} = -1.24$$

   and does not belong to the rejection region.

   Conclusion: we don't have enough evidence to reject the null hypothesis.

4. (25 points) The following data summarizes the proportional stress limits for specimens constructed using two different types of wood:

Type of wood	Sample size	Sample mean	Sample standard deviation
Red oak	14	8.48	0.79
Douglas fir	10	6.65	1.28

Assuming that both samples were selected from normal distributions, carry out a test of hypotheses with significance level $\alpha = 0.05$ to decide whether the true average proportional stress limit for red oak joints exceeds that for Douglas fir joints by more than 1 MPa. Provide the P-value of the test.

Denote the proportional stress limit for red oak and Douglas fir by μ_1 and μ_2. We are testing

$$H_0 : \mu_1 - \mu_2 = 1$$
$$H_a : \mu_1 - \mu_2 > 1$$

This is a two-sample t-test. Using the formulas in Chapter 10, we can compute t and the degrees of freedom:

$$t = 1.8181, df = 13.854$$

The p-value of the test is 0.045, which leads us to reject the null hypothesis.

5. (15 points) A data set $x_1, x_2, \ldots, x_{10}$ is sampled from a *log-normal distribution* with parameter μ and σ and satisfies

$$\sum x_i = 3860 \quad \text{and} \quad \sum x_i^2 = 4,574,802.$$

Given that for a lognormal distribution with parameter μ and σ, we have

$$E(X) = \exp\left(\mu + \frac{\sigma^2}{2}\right) \quad \text{and} \quad E(X^2) = \exp\left(2\mu + 2\sigma^2\right),$$

estimate μ and σ using the method of moments.

Using the method of moments, we have

$$\exp\left(\mu + \frac{\sigma^2}{2}\right) = E(X) = \bar{x} = \frac{\sum x_i}{n} = 386$$

and

$$\exp\left(2\mu + 2\sigma^2\right) = E(X^2) = \frac{\sum x_i^2}{n} = 457480.2.$$

Taking the logarithm of these equations, we obtain

$$\mu + \frac{\sigma^2}{2} = 5.96$$

and

$$2\mu + 2\sigma^2 = 13.03$$

Solving this system of equations, we obtain

$$\hat{\mu} = 5.41, \hat{\sigma}^2 = 1.11$$