MATH 350-011, Spring 2018 **Name (Print):** _____
Instructor: Vu Dinh
Homework 2
Due: Friday 03/02

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	30	
4	20	
5	10	
Total:	100	

1. (a) (10 points) A factory produces its entire output with three machines. Machines I, II, and III produce 50%, 30%, and 20% of the output, but 4%, 2%, and 4% of their outputs are defective, respectively. What fraction of the total output is defective?

   (b) (10 points) At a county fair carnival game there are 25 balloons on a board, of which 10 balloons are yellow, 8 are red, and 7 are green. A player throws darts at the balloons to win a prize and randomly hits one of them. Given that the first balloon hit is yellow, what is the probability that the next balloon hit is also yellow?

2. (a) (10 points) At a county fair carnival game there are 25 balloons on a board, of which 10 balloons are yellow, 8 are red, and 7 are green. A player throws darts at the balloons to win a prize and randomly hits one of them. What is the probability that the first two balloon hits are both yellow?

(b) (10 points) A red die and a white die are rolled. Let event

$$A = \{4 \text{ on the red die}\}$$

and event

$$B = \{\text{sum of dice is odd}\}.$$

Show that in this case

$$P(AB) = P(A)P(B).$$

3. (a) (10 points) A professor was meeting two students in Paris, one arriving by train from Amsterdam and the other arriving by train from Brussels at approximately the same time. Let A and B be the events that the respective trains are on time. Suppose we know from past experience that $P(A) = 0.93$, $P(B) = 0.89$, and $P(A \cap B) = 0.87$.
What is the probability that at least one train is on time?

(b) (20 points) A Pap smear is a screening procedure used to detect cervical cancer. For women with this cancer, there are about 16% false negatives; that is,

$$P[\text{test negative} \mid \text{patient has cancer}] = 0.16$$

and

$$P[\text{test positive} \mid \text{patient has cancer}] = 0.84.$$

For women without cancer, there are about 10% false positives; that is,

$$P[\text{test positive} \mid \text{patient does not have cancer}] = 0.10$$

and

$$P[\text{test negative} \mid \text{patient does not have cancer}] = 0.90.$$

In the United States, there are about 8 women in 100,000 who have this cancer. Assume that a woman is taking the test. Given that the test is positive, what is the probability that she has cervical cancer?

4. (a) (10 points) Suppose we have two hats: one has 4 red balls and 6 green balls, the other has 6 red and 4 green. We toss a fair coin, if heads, pick a random ball from the first hat, if tails from the second. What is the probability of getting a red ball?

   (b) (10 points) In a study conducted three years ago, 82% of the people in a randomly selected sample were found to have good financial credit ratings, while the remaining 18% were found to have bad financial credit ratings. Current records of the people from that sample show that 30% of those with bad credit ratings have since improved their ratings to good, while 15% of those with good credit ratings have since changed to having a bad credit rating. What percentage of people with good credit ratings now had bad ratings three years ago?

5. (a) (5 points) Let A, B be two events such that

$$P(AB) = P(A)P(B).$$

Show that

$$P(A^cB^c) = P(A^c)P(B^c).$$

(b) (5 points) Let E, F, G be events such that $P(E|F) \geq P(G|F)$ and $P(E|F^c) \geq P(G|F^c)$. Show that

$$P(E) \geq P(G).$$