MATH 350-011, Spring 2018 **Name (Print):** ____________________
Instructor: Vu Dinh
Practice exam
05/14/2018
Time Limit: 120 Minutes

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You are allowed to bring a one-sided A4-sized hand-written note as reference.
You may use calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Do not write in the table to the right.

Problem	Points	Score
1	40	
2	40	
3	40	
4	60	
5	20	
Total:	200	

1. An insurance company rents 35% of the cars for its customers from agency I and 65% from agency II. Assume that 8% of the cars of agency I and 5% of the cars of agency II break down during the rental periods.

   (a) (20 points) What is the probability that a car rented by this insurance company breaks down?

   (b) (20 points) Assume that a random selected car (rented by this insurance company) broke down, what is the probability that this car is from agency I?

2. (a) (20 points) A computer store has purchased three computers of a certain type at \$500 apiece. It will sell them for \$1000 apiece. The manufacturer has agreed to repurchase any computers still unsold after a specified period at \$200 apiece.

   Let X denote the number of computers sold, and suppose that

$$p(0) = 0.1, \ p(1) = 0.2, \ p(2) = 0.3, \ \text{and } p(3) = 0.4$$

   What is the expected profit of this computer store?

   (b) (20 points) When circuit boards used in the manufacture of compact disc players are tested, the long-run percentage of defectives is 5%. Let X be the number of defective boards in a random sample of size $n = 25$.

   Determine $P(X \leq 2)$, i.e., the probability of the event that there are less than or fewer than two defective boards in the sample.

3. The distribution of the amount of gravel (in tons) sold by a particular construction supply company in a given week is a continuous random variable X with pdf

$$f(x) = \begin{cases} c(1-x) & \text{if } 0 < x < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

where c is some unknown constant.

(a) (10 points) Compute c

(b) (20 points) Compute $P(1/4 \leq X \leq 3/4)$ and $Var(X)$

(c) (10 points) Find a such that $P[X \leq a] = \frac{15}{16}$

4. A nut company markets cans of deluxe mixed nuts containing almonds, cashews, and peanuts. Suppose the net weight of each can is exactly 1 lb, but the weight contribution of each type of nut is random. Because the three weights sum to 1, a joint probability model for any two gives all necessary information about the weight of the third type.

   Let X be the weight of almonds in a selected can and Y be the weight of cashews. Assume that the joint pdf for (X, Y) is

$$f(x,y) = \begin{cases} 24xy & \text{if } 0 \leq x \leq 1, \quad 0 \leq y \leq 1, \quad \text{and } 0 \leq x+y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) (20 points) Assume that 1 lb of almonds costs the company \$2.00, 1 lb of cashews costs \$3.00, and 1 lb of peanuts costs \$1.00. What is the expectation of the total costs of the contents of a can?

   (b) (20 points) Compute $f_X(x)$, $f_Y(y)$, $E(X)$, $E(Y)$

   (c) (20 points) Compute $Cov(X,Y)$ (i.e., the covariance between X and Y)

5. (a) (10 points) The loss due to a fire in a commercial building is modeled by a random variable X with density function

$$f(x) = \begin{cases} 0.005(20-x) & \text{if } 0 < x < 20 \\ 0 & \text{elsewhere.} \end{cases}$$

Given that a fire loss exceeds 8, calculate the probability that it exceeds 16.

(b) (10 points) Let A and B be two events and B^c be the complement of B.
Given that

$$P[A \cup B] = 0.7 \text{ and } P[A \cup B^c] = 0.9.$$

Compute $P(A)$.