Probability Theory and Simulation Methods

May 11th, 2018

Review

Final exam:

Monday, 5/21/2018.

Alison Hall Room 222

Overview

Week 1	Chapter 1: Axioms of probability
Week 2	Chapter 3: Conditional probability and independence
Week 4	Chapters 4, 6: Random variables
Week 9	Chapter 5, 7: Special distributions
Week 10	Chapters 8, 9, 10: Bivariate and multivariate distributions

Overview

1. Sample space and events
2. Axioms of probability

Chapter 3

1. Conditional Probability
2. Law of Multiplication
3. Law of Total Probability
4. Bayes' Formula
5. Independence

Chapter 6: Continuous random variables

Chapters 5 and 7

- Bernoulli distribution
- Binomial distribution
- Uniform distribution
- Normal distribution

Example 1

Example

Let X be a normal random variable with mean 39.8 and standard deviation 2.05. Compute

- $P[X \leq 40]$
- $P[X \geq 40]$
- $P[39.8 \leq X \leq 40]$

Table A.3 Standard Normal Curve Areas (*cont.*) $\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

Chapters 8, 9, 10

- Joint probability mass/density function
- Marginal probability mass/density function
- Conditional probability mass/density function
- Law of the unconscious statistician
- Covariance and correlation

Typical problem: bivariate

* Given a joint probability mass function between X, Y
* Compute:
  - Marginal pmf of X
  - $E[g(X,Y)]$ and $E[h(X)]$
  - $f_{X|Y}(x|y)$
  - $Cov(X,Y)$ and $\rho(X,Y)$
* For continuous random variables: using pdf instead of pmf
* The joint pmf is sometimes written as a function, sometimes represented as a table, so things might be confusing

Quiz 4

Let X and Y be two random variables with the following joint distribution

$$p(x,y) = \begin{cases} 0.1 & \text{if } (x,y) = (0,0) \\ 0.2 & \text{if } y = (0,1) \text{ or } (1,1) \\ 0.5 & \text{if } (x,y) = (1,0) \\ 0 & \text{elsewhere} \end{cases}$$

Compute

- $P[X \geq Y]$
- The marginal probability mass functions of X and Y
- Var(X) and Var(Y).

Quiz 5

Let X and Y be continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} x+y & \text{if } 0<x<1,\ 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$$

- Compute $E(X), E(Y), E(XY), \sigma_X, \sigma_Y$
- Compute $Cov(X,Y)$ and $\rho(X,Y)$
- Compute $Cov(1.5X+\sqrt{3}, -3Y+7.1)$