MATH 450: Mathematical statistics

Oct 31st, 2019

Lecture 19: Tests about a population mean

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Overview

9.1 Hypotheses and test procedures

- test procedures
- errors in hypothesis testing
- significance level
- 9.2 Tests about a population mean
- 9.4 P-values
- 9.3 Tests concerning a population proportion
- 9.5 Selecting a test procedure

Hypothesis testing

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In any hypothesis-testing problem, there are two contradictory hypotheses under consideration

- The null hypothesis, denoted by *H*₀, is the claim that is initially assumed to be true
- The alternative hypothesis, denoted by H_a , is the assertion that is contradictory to H_0 .

- H_0 will always be stated as an equality claim.
- If θ denotes the parameter of interest, the null hypothesis will have the form

$$H_0: \theta = \theta_0$$

- θ_0 is a specified number called the *null value*
- The alternative hypothesis will be either:

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$$H_a: \theta > \theta_0$$

- $H_a: \theta < \theta_0$
- $H_a: \theta \neq \theta_0$

A test procedure is specified by the following:

- A test statistic *T*: a function of the sample data on which the decision (reject *H*₀ or do not reject *H*₀) is to be based
- A rejection region \mathcal{R} : the set of all test statistic values for which H_0 will be rejected

The null hypothesis will then be rejected if and only if the observed or computed test statistic value falls in the rejection region, i.e., $T \in \mathcal{R}$

- A type I error consists of rejecting the null hypothesis H_0 when it is true
- A type II error involves not rejecting H_0 when H_0 is false.

Proposition

Suppose an experiment and a sample size are fixed and a test statistic is chosen. Then decreasing the size of the rejection region to obtain a smaller value of α results in a larger value of β for any particular parameter value consistent with H_a .

The approach adhered to by most statistical practitioners is

- \bullet specify the largest value of α that can be tolerated
- \bullet find a rejection region having that value of α rather than anything smaller
- α : the significance level of the test
- \bullet the corresponding test procedure is called a $\mathit{level}\;\alpha$ test

- Identify the parameter of interest
- Oetermine the null value and state the null hypothesis
- **③** State the appropriate alternative hypothesis
- Give the formula for the test statistic
- § State the rejection region for the selected significance level α
- Ompute statistic value from data
- Decide whether H_0 should be rejected and state this conclusion in the problem context

Normal population with known σ

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Null hypothesis

$$H_0: \mu = \mu_0$$

- The alternative hypothesis will be either:
 - $H_a: \mu > \mu_0$
 - $H_a: \mu < \mu_0$
 - $H_a: \mu \neq \mu_0$

Problem

The drying time of a certain type of paint under specified test conditions is known to be normally distributed with mean value 75 min and standard deviation 9 min. Chemists have proposed a new additive designed to decrease average drying time. It is believed that drying times with this additive will remain normally distributed with $\sigma = 9$. Because of the expense associated with the additive, evidence should strongly suggest an improvement in average drying time before such a conclusion is adopted. • Test of hypotheses:

$$egin{array}{l} H_{0}:\mu=75\ H_{a}:\mu<75 \end{array}$$

- n = 25. Rule: If $\bar{x} \leq c$, reject H_0 .
- Find the value of c to make this a level α test

$$\begin{aligned} \alpha &= P[\mathsf{Type \ I \ error}] \\ &= P[H_0 \ \text{is rejected while it is true}] \\ &= P[\bar{X} \leq c \ \text{while } \bar{X} \sim \mathcal{N}(75, 1.8^2)] \\ &= P\left[\frac{\bar{X} - 75}{1.8} \leq \frac{c - 75}{1.8}\right] \end{aligned}$$

• Rejection rule: $\bar{x} \leq 75 - 1.8 z_{\alpha}$

• To make it simpler, define $z = (\bar{x} - 75)/(1.8)$, then the rule is

$$z \leq -z_{\alpha}$$

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If we want to test

$$H_0: \mu = 75$$
$$H_a: \mu \neq 75$$

• *n* = 25. Rule: If

$$\bar{x} \leq -c$$
 or $x \geq c$

reject H_0 .

• Find the value of c to make this a level α test

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Normal population with known σ

Null hypothesis: $\mu = \mu_0$ Test statistic:

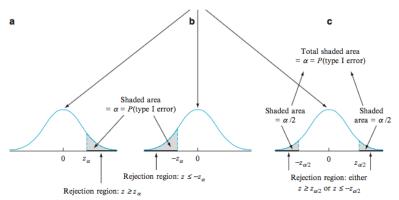
$$Z = \frac{X - \mu_0}{\sigma / \sqrt{n}}$$

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Alternative Hypothesis

Rejection Region for Level a Test

 $H_{a}: \mu > \mu_{0}$ $H_{a}: \mu < \mu_{0}$ $H_{a}: \mu \neq \mu_{0}$ $z \ge z_{\alpha}$ (upper-tailed test) $z \le -z_{\alpha}$ (lower-tailed test) either $z \ge z_{\alpha/2}$ or $z \le -z_{\alpha/2}$ (two-tailed test)



z curve (probability distribution of test statistic Z when H_0 is true)

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