MATH 450, Fall 2019 **Name (Print):** ______________________
Instructor: Vu Dinh
Practice problems
October 22nd, 2019
Time Limit: 50 Minutes

This exam contains 4 pages (including this cover page) and 3 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You are allowed to bring a one-sided A4-sized hand-written note as reference.
You may use calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Points	Score
1	30	
2	35	
3	35	
Total:	100	

Do not write in the table to the right.

1. Let X_1 and X_2 be the numbers of pounds of butterfat produced by two Holstein cows (one selected at random from those on the Koopman farm and one selected at random from those on the Vliestra farm, respectively) during the 305-day lactation period following the births of calves.

   Assume that the distribution of X_1 is $\mathcal{N}(693.2, 22820)$ and the distribution of X_2 is $\mathcal{N}(631.7, 19205)$. Moreover, let X_1 and X_2 be independent.

   (a) (20 points) What is the distribution of $T = X_1 - X_2$?

   (b) (10 points) Find $P(X_1 < X_2)$.

2. (a) (20 points) Let $\beta > 2$ and $X_1, \ldots, X_n$ be a random sample from a distribution with pdf

$$f(x) = \begin{cases} \frac{\beta-1}{x^\beta} & \text{if } x > 1 \\ 0 & \text{otherwise} \end{cases}$$

Derive the maximum-likelihood estimator for parameter β.

(b) (15 points) A data set $X_1, \ldots, X_{10}$ is sampled from a distribution with parameter α and β that satisfies

$$E(X) = \exp\left(\alpha + \frac{\beta^2}{2}\right), \qquad E(X^2) = \exp\left(2\alpha + 2\beta^2\right)$$

Assuming that

$$\sum_{i=1}^{10} x_i = 3860, \quad \text{and} \quad \sum_{i=1}^{10} x_i^2 = 4574802,$$

estimate α and β using the method of moments.

3. Let X equal the amount of orange juice (in grams per day) consumed by an American. Suppose it is known that the standard deviation of X is $\sigma = 16$. To estimate the mean μ of X, an orange growers association took a random sample of $n = 76$ Americans and found that they consumed, on the average, $\bar{x} = 133$ grams of orange juice per day.

   (a) (15 points) Construct a 90% confidence interval for μ.

   (b) (10 points) Find a 90% one-sided confidence interval for μ that provides an upper bound for μ.

   (c) (10 points) Suppose that X is approximately normal. Another single observation, X_{77}, is to be selected from the same distribution. Construct a 95% prediction interval for X_{77}.