MATH205, Fall 2021, Homework 3, Due Wednesday, Oct 20, 3:30 pm

Instructions

You can submit the homework either in paper or online

- Online: Take pictures of the written (theory) part; send them (along with the simulation part) to me on Slack or through Canvas before the lecture on Wednesday
- Paper: Print out the result of the simulation part and staple it with the written work; hand it in at the beginning of the lecture on Wednesday

1 Theory

1. Problem 1: Let X be a discrete random variable with the following probability mass function

$$p(x) = \begin{cases} \frac{x+1}{10}, & \text{for } x = 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

   - Compute $E[X]$ and $Var(X)$
   - Compute $E[2^X]$

2. Problem 2: Let X be a continuous random variable with the following probability density function

$$f(x) = \begin{cases} 3x^2, & \text{for } x \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

   - Compute $E[X]$ and $Var(X)$
   - Compute $P[0.25 \leq X \leq 0.75]$

3. Problem 3: Assume that the joint probability of X (receive values 1, 2) and Y (receives values 1, 2, 3) is represented by the following table

X \ Y	1	2	3
1	0.14	0.42	0.06
2	0.06	0.28	0.04

   - Are X and Y independent?
   - Compute $E(X + Y)$ and $E(XY)$.

2 Simulations

- Problem 4: Consider the probability distribution described in Problem 1.
  (a) Simulate a dataset of $n = 500$ random draws from the distribution. Compute the mean of the dataset.
  (b) Produce a bar plot of the dataset
  (c) Repeat part (a) with sample sizes

$$n = 500; 1000; 2000; 5000; 10000; 20000; 50000; 100000.$$

  For each case, compute the difference between the mean of the dataset and $E(X)$ (computed in Problem 1). Produce a plot of the difference vs. the sample size n.

- Problem 5: Consider the probability distribution described in Problem 2.
  (a) Simulate a dataset of $n = 500$ random draws from the distribution. Compute the mean of the dataset.
  (b) Produce a histogram of the sample
  (c) Repeat part (a) 5000 times. Use a vector v to save the mean of the dataset in the i^{th} replicate by $v[i]$. Produce a histogram of v.

Attach the scripts containing the commands and screenshots of the results of the scripts.