MATH205, Fall 2021, Homework 4, Due Wednesday, Nov 10, 3:30 pm

Instructions

You need to submit this homework online. Take pictures of the written (theory) part; send them (along with the simulation part) to me on Slack or through Canvas before the lecture on Wednesday.

1 Theory

1. Problem 1: A restaurant reports that the tip percentage at their restaurant has a mean value of 18% and a standard deviation of 6%. What is the approximate probability that the sample mean tip percentage for a random sample of 40 bills is between 16% and 20%?

2. Problem 2: The weight of a certain crab is known to have an expected value of 1.4 kg and standard deviation of 0.4 kg. The distribution is unknown. Consider one net that contains 112 crabs. What is the probability that the total weight of the crabs in the net is less than 150 kg?

3. Problem 3: Let X equal the amount of orange juice (in grams per day) consumed by an American. Suppose it is known that the standard deviation of X is $\sigma = 16$. To estimate the mean μ of X, an orange growers association took a random sample of $n = 76$ Americans and found that they consumed, on the average, $\bar{x} = 133$ grams of orange juice per day.

   - Construct a 90% confidence interval for μ.
   - Find a 90% one-sided confidence interval for μ that provides an upper bound for μ.

2 Simulations

Let X be a continuous random variable with the following probability density function

$$f(x) = \begin{cases} \frac{3}{2}x^2, & \text{for } x \in [-1, 1] \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute (on paper) the population mean $E(X)$

(b) Simulate a dataset of $n = 500$ random draws from the distribution. Construct the 95% confidence interval of the population mean from the dataset.

(c) Repeat part (b) $m = 100$ times. Compute the percentage of times (denoted by p) the constructed confidence interval contains $E(X)$

(d) Repeat part (c) with

$$m = 200; 500; 1000; 2000; 5000; 10000; 20000;$$

Produce a plot of the percentage p vs. the number of intervals m.