MATH 205: Statistical methods

Lab 8: Propagating uncertainty

Problem 1

- A farmer wants to know the area of his rectangular field. He asks two probabilists to measure the dimension of the field.
- They did and give him the following summary: Let X, Y be the width and the length of the rectangle, then X and Y are independent and

$$X \sim 30 + 3 * Uniform([0,1])$$

$$Y \sim 50 + 5 * Beta(2,5),$$

- Can you help the farmer find out the mean and the standard deviation of the area?

Problem 1

- Use two R functions rnorm and rexp to sample 20000 samples of (X, Y) and of the area of the field $A = X \times Y$.
- Compute the mean, the standard deviation and produce a histogram of A

Propagation of uncertainty

- In various problem, there is a quantity of interest Q, modeled as the output of a multivariate function, that is

$$Q = g(X_1, X_2, \dots, X_m)$$

  where $X_1, X_2, \dots, X_m$ are inputs that can be measured (with noise) and g is a known but complicated function.
- Central question: Assume that we know the distribution of $X_1, X_2, \dots, X_m$, can we make prediction about Q?
- Answer: Yes
  - Get random samples of $X_1, X_2, \dots, X_m$
  - Evaluate $Q = g(X_1, X_2, \dots, X_m) \rightarrow$ obtain samples of Q
  - The histogram of the dataset represent the distribution of Q

Black-box models

- In lots of examples, the function g is so complicated that you don't really know what it does. People refer to such cases as black-box predictions.
- As long as we can evaluate g, we can sample Q

Prey-predator model

- Lotka–Volterra equations
- Describes the dynamics of biological systems in which two species interact, one as a predator and the other as prey
- Assume that the two parameters Alpha and Beta in the codes are not constant, but follow the following distributions

$$Alpha, Beta \sim \mathcal{N}(10^{-3}, 10^{-8})$$

- We are interested in P, the number of preys at the end of the simulation
- Generate 2000 samples of P. Compute the mean, the standard deviation and produce a histogram of P.