MATH 205, Fall 2021
Instructor: Vu Dinh
Practice problem
October 25th, 2021
Time Limit: 75 Minutes

This exam contains 4 pages (including this cover page) and 3 problems. You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations/explanations will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Points	Score
1	30	
2	20	
3	30	
Total:	80	

1. The amount of time that a patient spends in a certain outpatient surgery center is a normal random variable with a mean value of 4.5 (hours) and a standard deviation of 2 (hours). Let $X_1, X_2, \ldots, X_{20}$ be the times for a random sample of 20 patients.

   (a) (10 points) What is the probability distribution of the sample mean

   $$\bar{X} = \frac{X_1 + X_2 + \ldots + X_{20}}{20}$$

   (b) (10 points) Compute $\mathbb{P}[\bar{X} \geq 5]$.

   (c) (10 points) Compute

   $$\mathbb{P}\left[\frac{X_1 + 4X_2}{5} \geq X_3\right].$$

2. (20 points) An insurance company examines its pool of auto insurance customers and gathers the following information:

    (i) All customers insure at least one car.

    (ii) 70% of the customers insure more than one car.

    (iii) 20% of the customers insure a sports car.

    (iv) Of those customers who insure more than one car, 15% insure a sports car.

    Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

3. (a) (15 points) Problem 1: Let X be a discrete random variable with the following probability mass function table

x	1	2	3
p(x)	0.2	0.3	0.5

Compute $Var[2^X]$.

(b) (15 points) Assume that the joint probability of X (receive values 1, 2) and Y (receives values 1, 2, 3) is represented by the following table

X \ Y	1	2	3
1	0.14	0.42	0.06
2	0.06	0.28	0.04

Compute $E(X^2Y)$.