MATH205, Fall 2022, Homework 3, Due Friday, Oct 21, 3:30 pm

Instructions

You can submit the homework either in paper or online

- Online: Take pictures of the written (theory) part; submit them (along with the simulation part) to me on Canvas before the lecture on Friday
- Paper: Print out the result of the simulation part and staple it with the written work; hand it in at the beginning of the lecture on Friday

Problems

1. Problem 1: You have a blood test for a rare disease that occurs by chance in 1 person in 100,000. If you have the disease, the test will report that you do with probability 0.90 (and that you do not with probability 0.1). If you do not have the disease, the test will report a false positive with probability 0.002.

   If the test says you do have the disease, what is the probability that you actually have the disease?

2. Problem 2: We throw two dice. Let

   $$\text{A be the event } \{\text{the sum of the points is } 7\},$$
   $$\text{B the event } \{\text{first die came up } 3\},$$
   $$\text{C the event } \{\text{second die came up } 4\}.$$

   (a) Prove that
   - A and B are independent
   - A and C are independent
   - B and C are independent

   (b) Compute $P[A|B \cap C]$

3. Problem 3: Let X be a continuous random variable with the following probability density function

   $$f(x) = \begin{cases} 3x^2, & \text{for } x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

   - Compute $P[0.25 \leq X \leq 0.75]$
   - Let $t \in (0,1)$, compute $P[X \leq t]$.

4. Problem 4: Let X be a discrete random variable with the following probability mass function table

x	1	2	3	4	5
p(x)	0.12	0.25	0.08	0.24	0.31

   - Compute $E[X]$ and $Var(X)$
   - Compute $E[2^X]$

5. Problem 5: Consider the probability distribution described in Problem 4.

   (a) Simulate a dataset of $n = 5000$ random draws from the distribution

   (b) Compute the mean, the median and the standard deviation of the dataset

   (c) Produce a bar plot of the dataset

   Attach the scripts containing the commands and screenshots of the results of the scripts. The plots should have clear titles and all axes labeled.