MATH205, Fall 2022, Homework 4. Due Friday, November 11th, 3:30 pm

Instructions

You can submit the homework either in paper or online

- Online: Take pictures of the written (theory) part; submit them (along with the simulation part) to me on Canvas before the lecture on Friday
- Paper: Print out the result of the simulation part and staple it with the written work; hand it in at the beginning of the lecture on Friday

Problems

1. Problem 1: Suppose the number of times a randomly selected customer of a large bank uses the bank's ATM during a particular period is a random variable with a mean value of 3.2 and a standard deviation of 2.4. For 100 randomly selected customers, how likely is it that the sample mean number of times the bank's ATM is used exceeds 4?

2. Problem 2: There are 40 students in an elementary statistics class. On the basis of years of experience, the instructor knows that the time needed to grade a randomly chosen first examination paper is a random variable with an expected value of 6 min and a standard deviation of 6 min.

   If grading times are independent and the instructor begins grading at 6:50 p.m. and grades continuously, what is the (approximate) probability that he finishes grading before the 11:00 p.m. TV news begins?

3. Problem 3: Let X equal the amount of orange juice (in grams per day) consumed by an American. Suppose it is known that the standard deviation of X is $\sigma = 16$. To estimate the mean μ of X, an orange growers association took a random sample of $n = 70$ Americans and found that they consumed, on the average, $\bar{x} = 133$ grams of orange juice per day.

   - Construct a 90% confidence interval for μ.
   - Find a 90% one-sided confidence interval for μ that provides an upper bound for μ.

4. Let X be a continuous random variable with the following probability density function

$$f(x) = \begin{cases} \frac{3}{8}x^2, & \text{for } x \in [0,2] \\ 0 & \text{otherwise} \end{cases}$$

   (a) Compute (on paper) the population mean $E(X)$

(b) Simulate a dataset of $n = 500$ random draws from the distribution. Construct the 95% confidence interval of the population mean from the dataset.

(c) Repeat part (b) $m = 100$ times. Compute the percentage of times (denoted by p) the constructed confidence interval contains $E(X)$

(d) Repeat part (c) with

$$m = 200; 500; 1000; 2000; 5000; 10000; 20000;$$

Produce a plot of the percentage p vs. the number of intervals m.