MATH 205: Statistical methods

Lecture 7: Sample space, events, and probability

Tentative schedule

Date	Theme/Topic	Labs	Assignments
Aug 31	Syllabus		
Sep 2—9	Chapter 1: Describing dataset	Section 2: Handling data	
Sep 12—16	Chapter 2: Looking at Relationships	Section 3: Univariate data	
Sep 19—23	Chapter 3: Basic Ideas in Probability	Section 4: Bivariate Data	Homework 1 (due 09/23)
Sep 26—30	Chapters 3-4	Section 4: Correlation	
Oct 3—7	Chapter 4: Random variables and expectations	Section 6: Random data	Homework 2 (due 10/07)
Oct 10—14	Chapter 5: Useful distributions	Section 7: The central limit theorem	
Oct 17—21	Chapter 6: Samples and populations	Section 9: Confidence interval estimation	Homework 3 (due 10/21)
Oct 24—28	Review Midterm exam		Midterm: Oct 28 (lecture) Oct 24-26 (labs)
Oct 31—Nov 4	Chapter 7: The significance of evidence	Section 10: Hypothesis testing	
Nov 7—11	Goodness of Fit	Section 12: Goodness of Fit	Homework 4 (due 11/11)
Nov 14—18	Linear Regression	Section 13: Linear regression	
Nov 21–25	Thanksgiving break		
Nov 28 —Dec 2	One-Way Analysis of Variance	Section 15: Analysis of variance	Homework 5 (due 12/02)
Dec 5—7	Selected topics + Review		
Exam week			

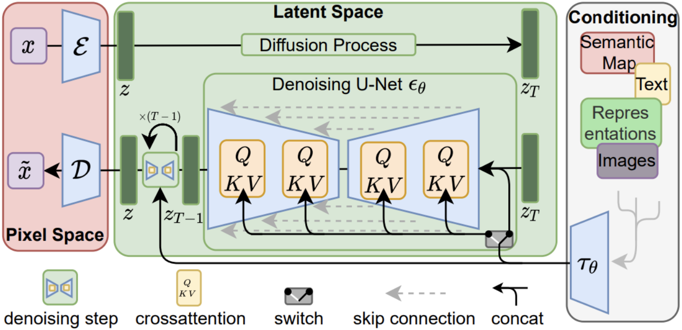
Topics

- Sample space and events
- Basic properties of probability
- Advanced properties of probability
- Compute probability
  - Computing event probabilities by counting outcomes
  - Computing probabilities by reasoning about sets

Stable diffusion

Midjourney

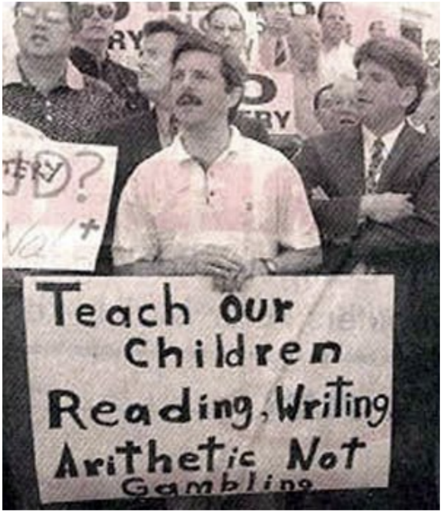
Latent diffusion



Probability and gambling

Modern probability started when Pascal and Fermat discussed gambling

Against teaching gambling



How to gamble (according to mathematicians)

1. Consider all possible outcomes
2. Assess how likely each outcome will happen
3. Choose the course of action that benefits you the most
4. Profit!

Sample space and events

1. An experiment: is any action, process, or phenomenon whose outcome is subject to uncertainty
2. An outcome: is a result of an experiment
   Each run of the experiment results in one outcome
3. A sample space: is the set of all possible outcomes of an experiment
4. An event: is a subset of the sample space.
   An event occurs when one of the outcomes that belong to it occurs

Sample space and events: example

1. Experiment: Toss a coin
2. Outcome: either head (H) or tail (T)
3. Sample space: $\{H, T\}$
4. Events: $\{H, T\}$, $\{H\}$, $\{T\}$, $\emptyset$

Sample space and events: example

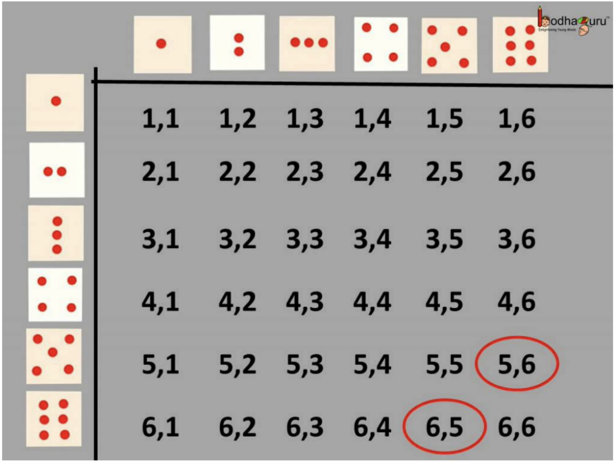
1. Experiment: Toss a coin 2 times
2. Sample space: $\{HH, HT, TH, TT\}$
3. Events: There are 16 different events. Examples:
   - E_1 = the result of the two tosses are different $= \{HT, TH\}$
   - E_2 = the result of the second toss is head$= \{HH, TH\}$

Sample space and events: example

1. Experiment: Toss a regular dice
2. Sample space: $\{1, 2, 3, 4, 5, 6\}$
3. Some events
   - E_1 = the result is an even number = $\{2, 4, 6\}$
   - E_2 = the result is greater than 2= $\{3, 4, 5, 6\}$
   - $E_3 = \{1, 3, 5, 6\}$

Sample space and events: example

1. Experiment: Toss two regular dice
2. Event E_1 = the summation of the two dice is 11



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Define probability

1. Experiment: Toss a FAIR coin
2. Outcome: either head (H) or tail (T), each with probability 0.5
3. Sample space: $\{H, T\}$
4. Events: $\{H, T\}$, $\{H\}$, $\{T\}$, $\emptyset$
5. Define:

$$P[\{H\}] = P[\{T\}] = 1/2, P(\emptyset) = 0, P(\{H, T\}) = 1$$

Define probability

1. Experiment: Toss a FAIR coin TWO times
2. Outcome:

$$P(\{HH\}) = P(\{HT\}) = P(\{TH\}) = P(\{TT\}) = 1/4$$

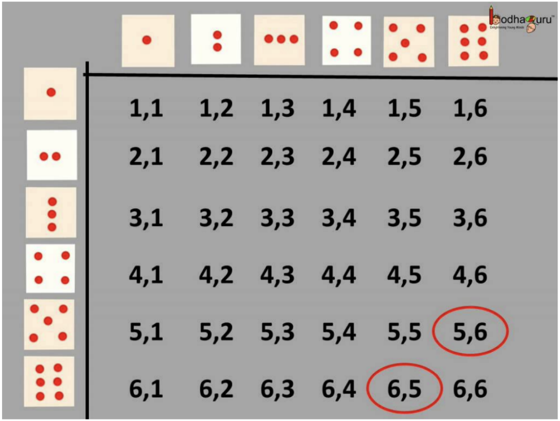
3. Sample space: $\{HH, HT, TH, TT\}$
   - E_1 = the results of the two coins are different = $\{HT, TH\}$
   - E_2 = the result of the second coin is head= $\{HH, TH\}$
4. Thus

$$P[E_1] = P(\{HT\}) + P(\{TH\}) = 1/2$$

Define probability

1. Experiment: Toss two regular dice
2. E_1 = the summation of the two dice is 11

$$P[E_1] = 1/18$$



Some fundamental questions

Probability is a function defined on the set of events of an experiment

1. What conditions should we impose to define probability?

Different views of probability

- Frequentist: The probability of an outcome is the frequency of that outcome in a very large number of repeated experiments
- Bayesian: Probability is a quantification of a belief about how often an outcomes occurs

Different probabilities

- Given an experiment and a sample space, we can define many different probabilities
- Experiment: tossing a coin, $\Omega = \{H, T\}$
- If you believe the coin is fair:

$$P(\emptyset) = 0, \quad P(\{H\}) = 0.5, \quad P(\{T\}) = 0.5, \quad P(\{H, T\}) = 1.$$

- If you do not, then maybe

$$P(\emptyset) = 0, \quad P(\{H\}) = 0.7, \quad P(\{T\}) = 0.3, \quad P(\{H, T\}) = 1.$$