MATH 205: Statistical methods

Lecture 10: Compute probability by reasoning about sets

Topics

- Sample space and events
- Basic properties of probability
- Advanced properties of probability
- Compute probability
  - Computing event probabilities by counting outcomes
  - Computing probabilities by reasoning about sets

Sample space and events

1. An experiment: is any action, process, or phenomenon whose outcome is subject to uncertainty
2. An outcome: is a result of an experiment
   Each run of the experiment results in one outcome
3. A sample space: is the set of all possible outcomes of an experiment
4. An event: is a subset of the sample space.
   An event occurs when one of the outcomes that belong to it occurs

Basic properties of probability

Useful Facts 3.1 (Basic Properties of the Probability Events)
We have

- The probability of every event is between zero and one; in equations
$$0 \le P(\mathcal{A}) \le 1$$
for any event $\mathcal{A}$.
- Every experiment has an outcome; in equations,
$$P(\Omega) = 1.$$
- The probability of disjoint events is additive; writing this in equations requires some notation. Assume that we have a collection of events $\mathcal{A}_i$, indexed by i. We require that these have the property $\mathcal{A}_i \cap \mathcal{A}_j = \varnothing$ when $i \neq j$. This means that there is no outcome that appears in more than one $\mathcal{A}_i$. In turn, if we interpret probability as relative frequency, we must have that
$$P(\cup_i \mathcal{A}_i) = \sum_i P(\mathcal{A}_i)$$

Advanced properties of probability

Useful Facts 3.2 (Properties of the Probability of Events)

- $P(\mathcal{A}^c) = 1 - P(\mathcal{A})$
- $P(\varnothing) = 0$
- $P(\mathcal{A} - \mathcal{B}) = P(\mathcal{A}) - P(\mathcal{A} \cap \mathcal{B})$
- $P(\mathcal{A} \cup \mathcal{B}) = P(\mathcal{A}) + P(\mathcal{B}) - P(\mathcal{A} \cap \mathcal{B})$

- If $A \subset B$, then $P(A) \leq P(B)$.
- For any events A, B

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

Computing probabilities by reasoning about sets

Problem

You throw two fair six-sided dice, and add the number of spots. What is the probability of getting a number divisible by 2, but not by 5?

Computing probabilities by reasoning about sets

Problem

You throw two fair six-sided dice, and add the number of spots. What is the probability of getting a number divisible by either 2 or 5, or both?

Computing probabilities by reasoning about sets

Problem

Suppose that in a community of 400 adults, 300 bike or swim or do both, 160 swim, and 120 swim and bike. What is the probability that an adult, selected at random from this community, bikes?