MATH 205: Statistical methods

Lecture 11: Independence

Tentative schedule

Date	Theme/Topic	Labs	Assignments
Aug 31	Syllabus		
Sep 2—9	Chapter 1: Describing dataset	Section 2: Handling data	
Sep 12—16	Chapter 2: Looking at Relationships	Section 3: Univariate data	
Sep 19—23	Chapter 3: Basic Ideas in Probability	Section 4: Bivariate Data	Homework 1 (due 09/23)
Sep 26—30	Chapters 3-4	Section 4: Correlation	
Oct 3—7	Chapter 4: Random variables and expectations	Section 6: Random data	Homework 2 (due 10/07)
Oct 10—14	Chapter 5: Useful distributions	Section 7: The central limit theorem	
Oct 17—21	Chapter 6: Samples and populations	Section 9: Confidence interval estimation	Homework 3 (due 10/21)
Oct 24—28	Review Midterm exam		Midterm: Oct 28 (lecture) Oct 24-26 (labs)
Oct 31—Nov 4	Chapter 7: The significance of evidence	Section 10: Hypothesis testing	
Nov 7—11	Goodness of Fit	Section 12: Goodness of Fit	Homework 4 (due 11/11)
Nov 14—18	Linear Regression	Section 13: Linear regression	
Nov 21–25	Thanksgiving break		
Nov 28 —Dec 2	One-Way Analysis of Variance	Section 15: Analysis of variance	Homework 5 (due 12/02)
Dec 5—7	Selected topics + Review		
Exam week			

Chapter 3: Basic ideas in probability

- Experiments ,outcomes, events, and probability.
- Independence
- Conditional probability

Independence

Independence

- Some experimental results do not affect others
- Example: if I flip a coin twice, whether I get heads on the first flip has no effect on whether I get heads on the second flip
- We refer to events with this property as independent.

Independence

Definition

Two events A and B are independent if and only if

$$P(A \cap B) = P(A)P(B)$$

Dependent events: example

Toss a fair dice:

- A: the event that the die comes up with an odd number of spots
- B: the event that the number of spots is larger than 3.
- $P(A) = P(B) = 1/2$
- If we know that A has occurred, then we know the die shows either 1, 3, or 5 spots. One of these outcomes belongs to B, and two do not. $P(A \cap B) = 1/6$.
- This means that knowing that A has occurred tells you something about whether B has occurred.

$\rightarrow$ These events are interrelated.

Independence: example

Problem

A red die and a white die are rolled. Let event

$$A = \{4 \text{ on the red die}\}$$

and event

$$B = \{\text{sum of dice is odd}\}.$$

Show that A and B are independent.

Independence

Problem

Prove that if A and B are independent, then A and B^c are independent as well.

Independence

Problem

Prove that if A and B are mutually exclusive events, and $P(A) > 0, P(B) > 0$, then they are dependent.

Independence

Problem

If $P(A) = 0.5$, $P(B) = 0.2$ *and* $P(A \cup B) = 0.65$. *Are* A *and* B *independent?*

Independence

Problem

I search a DNA database with a sample. Each time I attempt to match this sample to an entry in the database, there is a probability of an accidental chance match of 10^{-4}*. Chance matches are independent. There are 20,000 people in the database. What is the probability I get at least one match, purely by chance?*