MATH 205: Statistical methods

Lecture 19: Linear combinations of random variables

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Multivariate distributions

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Joint probability

Definition 4.4 (Joint Probability Distribution of Two Discrete Random Variables) Assume we have two random variables X and Y. The probability that X takes the value x and Y takes the value y could be written as $P({X = x} \cap {Y = y})$. It is more usual to write it as

P(x, y).

This is referred to as the **joint probability distribution** of the two random variables (or, quite commonly, the **joint**). You can think of this as a table of probabilities, one for each possible pair of *x* and *y* values.

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Marginal probability

Definition 4.6 (Marginal Probability of a Random Variable) Write P(x, y) for the joint probability distribution of two random variables X and Y. Then

$$P(x) = \sum_{y} P(x, y) = \sum_{y} P(\{X = x\} \cap \{Y = y\}) = P(\{X = x\})$$

is referred to as the marginal probability distribution of X.

Independent variables

Definition 4.7 (Independent Random Variables) The random variables X and Y are **independent** if the events $\{X = x\}$ and $\{Y = y\}$ are independent for all values x and y. This means that

$$P(\{X = x\} \cap \{Y = y\}) = P(\{X = x\})P(\{Y = y\}),$$

which we can rewrite as

P(x, y) = P(x)P(y)

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Example

Example

Measurements for the length and width of rectangular plastic covers for CDs are rounded to the nearest mm. Let X denote the length and Y denote the width. Assume that the joint probability of X and Y is represented by the following table

$$\begin{array}{c|c} x = length \\ \hline 129 & 130 & 131 \\ y = width & 15 & 0.12 & 0.42 & 0.06 \\ 16 & 0.08 & 0.28 & 0.04 \\ \end{array}$$

Are X and Y independent?

Expected value: multivariate

Given two discrete random variable X, Y and f is a function of (X, Y). Then f(X, Y) is a also discrete random variable, and

$$\mathbb{E}[f(X,Y)] = \sum_{x,y} f(x,y) P(x,y)$$

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Definition The covariance of of two random variables X and Y is

$$cov(X, Y) = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$$

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Covariance: properties

Note that

$$cov(X,X) = var(X)$$

• The covariance of of two random variables X and Y can be computed as

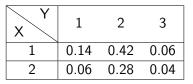
$$cov(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

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Example

Example

Assume that the joint probability of X (receive values 1, 2) and Y (receives values 1, 2, 3) is represented by the following table



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Compute Cov(X, Y).

Independent variables have zero covariance

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Proposition

If X and Y are independent, then

- $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$
- cov(X, Y) = 0

Variance of sum of independent variables

Proposition If X and Y are independent, then

$$var(X + Y) = var(X) + var(Y)$$

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Exercise

Problem

A random variable X has the following pmf table

Let Y be a continuous r.v. with density function

$$f(y) = egin{cases} 2x & \textit{if } y \in [0,1] \ 0 & \textit{otherwise} \end{cases}$$

Assume that X and Y are independent. Compute E(X + Y), Var(X + Y)

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