MATH 205: Statistical methods

Lecture 21: Useful distributions (cont.)

Announcements

- Homework 3 due this Friday, before lecture
- Quiz 3 this Friday (end of lecture)
- Next week
  - Monday–Wednesday: Midterm-Simulation in the labs.
    Open book + internet.
  - Wednesday's lecture: Review session before the written exam
  - Friday: Midterm-Written.
    Closed book.
    Can use calculator.
    Can bring a A4-sized hand-written one-sided note to the exam.

Review: Random variables and expectations

4.1 Random variables and probability distribution

- Discrete
- Continuous
- Joint and marginal distributions
- Independent variables

4.2 Expectations

- Mean
- Variance
- Covariance
- Expectations of functions of random variable(s)

Linear combination of random variables

Theorem

Let $X_1, X_2, \ldots, X_n$ be independent random variables (with possibly different means and/or variances). Define

$$T = a_1 X_1 + a_2 X_2 + \ldots + a_n X_n$$

then the mean and the standard deviation of T can be computed by

- $E(T) = a_1 E(X_1) + a_2 E(X_2) + \ldots + a_n E(X_n)$
- $Var(T) = a_1^2 Var(X_1) + a_2^2 Var(X_2) + \ldots + a_n^2 Var(X_n)$

Chapter 5: Useful distributions

Useful distributions

In the lectures:

- Uniform distribution
- Normal distribution
- Bernoulli distribution
- Binomial distribution

In the lab:

- Geometric distribution
- Poisson distribution
- Beta distribution
- Gamma distribution
- Exponential distribution

The Discrete Uniform Distribution

Definition

A random variable has the discrete uniform distribution if it takes each of k values with the same probability $1/k$, and all other values with probability zero.

Problem

Consider a random variable X that follows discrete uniform distribution on the set $\{1, 2, 3, 4\}$.
Compute $E(X)$ and $Var(X)$.

The continuous uniform distribution

Definition

Write l for the lower bound and u for the upper bound. The probability density function for the uniform distribution on the interval l, u is

$$f(x) = \begin{cases} \frac{1}{u-l}, & x \in [l, u] \\ 0 & otherwise \end{cases}$$

Problem

Consider a random variable X that follows continuous uniform distribution on $[l, u]$.
Compute $E(X)$ and $Var(X)$ (in terms of l, u).

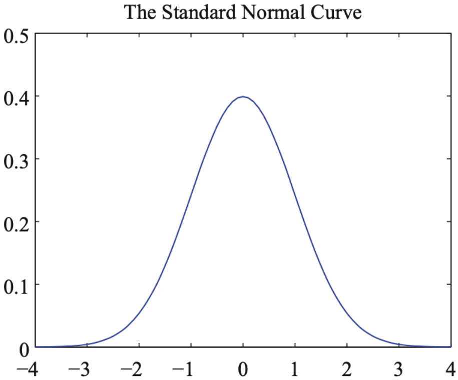
The standard normal distribution

Definition

The probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

is known as the standard normal distribution.



The standard normal distribution has mean 0 and variance 1.

Normal distributions

Write μ for the mean of a random variable X and σ for its standard deviation; if

$$\frac{X - \mu}{\sigma}$$

has a standard normal distribution, then X is a normal random variable.

Normal distributions

Definition

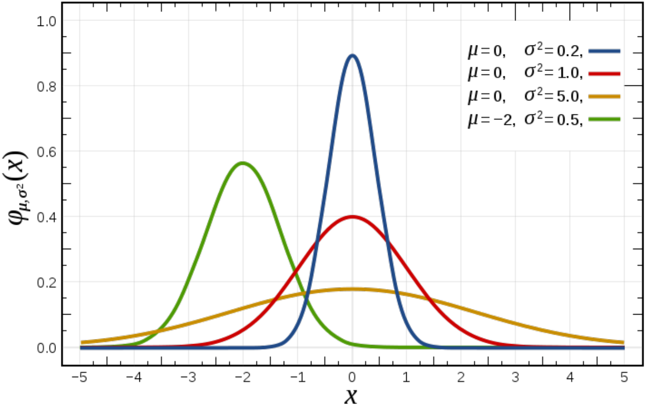
If X a normal random variable with probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

then

$$E[X] = \mu, \qquad Var(X) = \sigma^2$$

$\mathcal{N}(\mu, \sigma^2)$



$$E(X) = \mu, Var(X) = \sigma^2$$

Shifting and scaling normal random variables

Problem

Let X be a normal random variable with mean μ and standard deviation σ.
Then

$$Z = \frac{X - \mu}{\sigma}$$

follows the standard normal distribution.

Exercise

Problem

Let X be a $\mathcal{N}(3, 9)$ random variable. Compute $P[X \leq 5.25]$.