

MATH 205, Fall 2022
Instructor: Vu Dinh
Midterm practice
October 26th, 2021
Time Limit: 50 Minutes

This exam contains 4 pages (including this cover page) and 3 problems.
You are allowed to bring a one-sided A4-sized hand-written note as reference.
You may use calculators.

Problem	Points	Score
1	15	
2	15	
3	30	
Total:	60	

1. (15 points) Consider a the distribution with the following probability mass function

x	30	35	40
p(x)	0.2	0.3	0.5

Let X_1, X_2 be two independent random sample from this distribution, and $T = X_1 + 2X_2$.

- Compute the expected value and the standard deviation of X_1 .
- Compute the expected value and the standard deviation of T .

2. (15 points) An actuary studying the insurance preferences of automobile owners makes the following conclusions:
- An automobile owner is twice as likely to purchase collision coverage as disability coverage.
 - The event that an automobile owner purchases collision coverage is independent of the event that they purchases disability coverage.
 - The probability that an automobile owner purchases both collision and disability coverages is 0.15.

Calculate the probability that an automobile owner purchases disability coverage.

3. (a) (15 points) Assume that the joint probability of X (receive values 1, 2) and Y (receives values 1, 2, 3) is represented by the following table

X \ Y	1	2	3	4
1	0.15	0.32	0.06	0.1
2	0.05	0.18	0.04	0.1

Construct the probability mass function table of Y and compute $Var[Y^2]$.

- (b) (15 points) Let X be a continuous random variable with the following probability density function

$$f(x) = \begin{cases} \frac{3}{2}x^2, & \text{for } x \in [-1, 1] \\ 0 & \text{otherwise} \end{cases}$$

Compute

- $E(X^2 + X)$
- Compute $P[-0.5 \leq X \leq 0.75]$