

Random variable  $X$  with density

$$f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Preparation.

① Compute the distribution function

$$t < 0 \quad F(t) = P[X \leq t] = 0$$

$$t \geq 0$$

$$\begin{aligned} F(t) = P[X \leq t] &= \int_0^t 2e^{-2x} dx \\ &= -e^{-2x} \Big|_0^t \\ &= -e^{-2t} - (-e^{-2 \cdot 0}) \\ &= -e^{-2t} + 1 \end{aligned}$$

②  $u \in (0, 1)$

$$\text{Solve:} \quad F(x) = u$$

$$1 - e^{-2x} = u$$

$$1 - u = e^{-2x}$$

$$\ln(1 - u) = -2x$$

$$x = -\frac{1}{2} \ln(1 - u)$$

Density function:

$$f(x) = \begin{cases} 2x & x \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

① Compute the distribution function

$$F(t) = P[X \leq t]$$

$$t < 0 \quad F(t) = 0$$

$$t > 1 \quad F(t) = 1$$

$$t \in (0, 1) \quad F(t) = \int_0^t 2x \, dx = x^2 \Big|_0^t = t^2 - 0^2 = t^2$$

② Solve the equation

$$u \in (0, 1)$$

$$F(x) = u$$

$$x^2 = u$$

$$x = \sqrt{u}$$