

Question 3:

(a)

y	1	2	3	4
p(y)	0.2	0.5	0.1	0.2

$$E(Y^2) = 1^2 \times 0.2 + 2^2 \times 0.5 + 3^2 \times 0.1 + 4^2 \times 0.2 = 6.3$$

$$E(Y^4) = 1^4 \times 0.2 + 2^4 \times 0.5 + 3^4 \times 0.1 + 4^4 \times 0.2 = 67.5$$

$$\text{Var}(Y^2) = E[(Y^2)^2] - [E(Y^2)]^2$$

$$= E(Y^4) - [E(Y^2)]^2$$

$$= 27.81$$

(b)

$$E[X^2 + X] = \int_{-1}^1 (x^2 + x) \frac{3}{2} x^2 dx$$

$$= \frac{3}{2} \int_{-1}^1 (x^4 + x^3) dx$$

$$= \frac{3}{2} \left[ \frac{x^5}{5} + \frac{x^4}{4} \right]_{-1}^1$$

$$= \frac{3}{2} \left( \frac{2}{5} \right) = \frac{3}{5}$$

$$P[-0.5 \leq x \leq 0.75] = \int_{-0.5}^{0.75} \frac{3}{2} x^2 dx$$

$$= \left. \frac{x^3}{2} \right|_{-0.5}^{0.75}$$

$$= 0.273$$