MATH 205, Fall 2022
Instructor: Vu Dinh
Practice exam
December 7th, 2022

Name (Print): ______________________

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You are allowed to bring a one-sided A4-sized hand-written note as reference.
You may use calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Do not write in the table to the right.

Problem	Points	Score
1	40	
2	40	
3	40	
4	40	
5	40	
Total:	200	

1. Consider a the distribution with the following probability mass function

x	30	35	40
p(x)	0.2	0.3	0.5

Let X_1, X_2 be two independent random sample from this distribution, and $T = X_1 - X_2$.

(a) (10 points) Compute the expected value and the standard deviation of X_1.

(b) (10 points) Compute the expected value and the standard deviation of T.

(c) (20 points) Compute

$$P[T = 5]$$

2. (a) (20 points) Let Y be a continuous random variable with the following probability density function

$$f(y) = \begin{cases} \frac{3}{7}y^2, & \text{for } \ y \in [1,2] \\ 0 & \text{otherwise} \end{cases}$$

Compute $P[1.25 \leq Y \leq 1.75]$.

(b) (20 points) Assume that the joint probability of X (receive values 0, 1) and Y (receives values 0, 1, 2) is represented by the following table

X \ Y	0	1	2
0	1/18	1/18	4/18
1	2/18	5/18	5/18

Are X and Y independent?

3. (a) (20 points) Of the patients in a hospital, 20% of those with, and 35% of those without myocardial infarction have had strokes. If 40% of the patients have had myocardial infarction, what percent of the patients have had strokes?

   (b) (20 points) In a region, the correlation coefficient between corn yield and peanut yield (planted in the same soil, in MT/ha) is 0.9. We also know that

   - The mean and the standard deviation of corn yield is 3.2 and 2.4, respectively.
   - The mean and the standard deviation of peanut yield is 1.8 and 0.72, respectively.

   Using this information, predict the expected corn yield of a filed with peanut yield of 1.76.

4. Urban storm water can be contaminated by many sources, including discarded batteries. When ruptured, these batteries release metals of environmental significance. The paper "Urban Battery Litter" (J. Environ. Engr., 2009: 46–57) presented summary data for characteristics of a variety of batteries found in urban areas around Cleveland. A sample of 51 Panasonic AAA batteries gave a sample mean zinc mass of 2.06 (g) and a sample standard deviation of 0.141 (g).

   (a) (20 points) With a significance level of $\alpha = 0.001$, does this data provide compelling evidence for concluding that the population mean zinc mass exceeds 2.0 g.?

   (b) (20 points) Construct a 88% confidence interval of the population mean zinc mass.

5. A study was carried out in an attempt to improve student performance in a low-level university mathematics course. The study involved assigning the students to sections based on odd or even Social Security number. Half of the sections were taught traditionally, whereas the other half were taught in a way that hopefully would keep the students involved.

   The final exam scores for the 79 students taught traditionally (the control group) and for the 85 students taught with more involvement (the experimental group) are summarized as follows:

Group	Sample Size	Sample Mean	Sample SD
Control	79	23.87	11.60
Experimental	85	27.34	8.85

   (a) (20 points) With a significance level of $\alpha = 0.05$, does this data provide compelling evidence that that the experimental method of instruction is an improvement?

   (b) (20 points) Construct a 92% confidence interval of the true average difference in performance of the two approaches ($\mu_{experimental} - \mu_{control}$).