MATH 637 – Homework 4

Due: November 10th, 11:59 PM

Submit your solutions to Canvas as a PDF file. You may scan (or take a good picture of) a handwritten document, but they will be returned ungraded if they are not legible.

You can use any results that have been stated/proven in class.

Question 1 (2%)

Let X be a non-negative random variable with $E[e^X] \leq 10$. Show that

$$P[X \geq 10] < 0.001$$

Question 2 (3%)

Let X be a non-negative random variable with $E[\ln(X)] = 2$. Show that

$$P[X \geq 1000] < 0.3$$

Question 3 (3%)

Let $X_1, X_2, \ldots, X_{100}$ be i.i.d copies of a random variable $X \in [-1, 1]$ and $E[X] = 0$. Denote

$$S = X_1 + X_2 + \ldots + X_{100}$$

Show that

$$P\left[|S| \geq 60\right] < 0.05$$

Question 4 (2%)

Let $X_1, X_2, \ldots, X_{100}$ be i.i.d copies of the standard normal random variables $\mathcal{N}(0, 1)$. Denote

$$S = X_1 + X_2 + \ldots + X_{100}$$

Compute

$$P\left[|S| \geq 20\right]$$