MATH450 **Name (Print):** ____________________
Instructor: Vu Dinh
Final examination
12/14/2018
Time Limit: 120 Minutes

This exam contains 8 pages: this cover page, 5 problems and 2 tables. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books/notes on this exam. You may use calculator. You are allowed to bring a one-sided A4-sized hand-written note as reference.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Do not write in the table to the right.

Problem	Points	Score
1	40	
2	40	
3	40	
4	40	
5	40	
Total:	200	

1. Let X_1 and X_2 equal the number of pounds of butterfat produced by two Holstein cows (one selected at random from those on the Koopman farm and one selected at random from those on the Vliestra farm, respectively) during the 305-day lactation period following the births of calves.

   Assume that the distribution of X_1 is $\mathcal{N}(693.2, 22820)$ and the distribution of X_2 is $\mathcal{N}(631.7, 19205)$. Moreover, let X_1 and X_2 be independent.

   (a) (20 points) What is the distribution of $T = X_1 - X_2$?

   (b) (20 points) Find $P(X_1 > X_2)$.

2. Let $0 < \theta < \infty$ and $X_1, X_2, \ldots, X_n$ be a random sample from a distribution that has pdf

$$f(x;\theta) = \frac{1}{\theta} x^{1/\theta - 1}, \quad 0 < x < 1$$

(a) (20 points) Construct the maximum likelihood estimator $\hat{\theta}_{ML}$ of θ.

(b) (20 points) Show that $E[\hat{\theta}_{ML}] = \theta$ and thus that $\hat{\theta}_{ML}$ is an unbiased estimator of θ. (Hint: First, compute $E[\ln(X_i)]$ using the law of the unconscious statistician)

3. (40 points) Let X and Y denote the tarsus lengths of male and female grackles, respectively. Assume that X is $\mathcal{N}(\mu_1, \sigma_1^2)$ and Y is $\mathcal{N}(\mu_2, \sigma_2^2)$.

   The collected data is as follows. For male grackles, $m = 80$ measurements are recorded, with sample mean 28.25 and sample standard deviation 6.5. For female grackles, $n = 74$ measurements are taken, with sample mean 25.88 and sample standard deviation 9.18.

   Test the null hypothesis $H_0 : \mu_1 = \mu_2$ against $H_a : \mu_1 \neq \mu_2$ with level of significance $\alpha = 0.05$. Provide the p-value of the test.

4. In attempting to control the strength of the wastes discharged into a nearby river, a paper firm has taken a number of measures on $n = 20$ consecutive days, for which the observed values of the sample mean and sample standard deviation were $\bar{x} = 308.8$ and $s = 115.15$ (measured in parts per million of permanganate). We assume that the strength of the wastes discharged by the firm follows the normal distribution.

   (a) (20 points) Construct the one-sided 99% upper confidence interval for μ

   (b) (20 points) Members of the firm believe that they have reduced the oxygen-consuming power of their wastes from a previous mean μ of 500.

   Carry out a test of hypotheses with significance level $\alpha = 0.01$ to decide whether the collected data support this belief.

5. (a) (20 points) Trace metals in drinking water affect the flavor and an unusually high concentration can pose a health hazard. Ten pairs of data were taken measuring zinc concentration in bottom water and surface water:

Location	1	2	3	4	5	6	7	8	9	10
Bottom	.430	.266	.567	.531	.707	.716	.651	.589	.469	.723
Surface	.415	.238	.390	.410	.605	.609	.632	.523	.411	.612

Using significance level $\alpha = 0.05$ and assuming that the data follows normal distribution, does the data suggest that the true average concentration in the bottom water exceeds that of surface water?

(b) (20 points) Suppose that for a parameter θ, X is the outcome of the roll of a four-sided tetrahedral die with the following probability mass function

x	1	2	3	4
p(x)	$\frac{3\theta}{4}$	$\frac{\theta}{4}$	$\frac{3(1-\theta)}{4}$	$\frac{(1-\theta)}{4}$

Suppose the die is rolled 10 times with outcomes

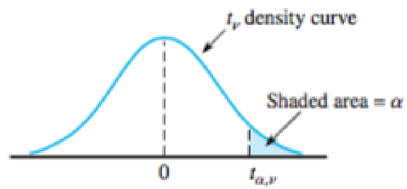
$$4, 1, 2, 3, 1, 2, 3, 4, 2, 3$$

Use the method of moments to obtain an estimator of θ.

Table A.3 Standard Normal Curve Areas (*cont.*) $\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table A.5 Critical Values for t Distributions



	α						
ν	.10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659