Mathematical statistics

April 29th, 2019

Lecture 26: P-values

- Final exam:

  Wednesday, 5/29/2019, Wednesday, 10:30am –12:30pm
  Ewing Hall Room 101

- Course evaluation

- Last homework due next Friday

Overview

Key steps in statistical inference

- Understand statistical models [Chapter 6]
- Come up with reasonable estimates of the parameters of interest [Chapter 7]
- Quantify the confidence with the estimates [Chapter 8]
- Testing with the parameters of interest [Chapter 9]

Contexts

- The central mega-example: population mean μ
- Difference between two population means

Hypothesis testing

In any hypothesis-testing problem, there are two contradictory hypotheses under consideration

- The null hypothesis, denoted by H_0, is the claim that is initially assumed to be true

- The alternative hypothesis, denoted by H_a, is the assertion that is contradictory to H_0.

Implicit rules (of this chapter)

- H_0 will always be stated as an equality claim.
- If θ denotes the parameter of interest, the null hypothesis will have the form

$$H_0 : \theta = \theta_0$$

- θ_0 is a specified number called the *null value*
- The alternative hypothesis will be either:
  - $H_a : \theta > \theta_0$
  - $H_a : \theta < \theta_0$
  - $H_a : \theta \neq \theta_0$

Test procedures

A test procedure is specified by the following:

- A test statistic T: a function of the sample data on which the decision (reject H_0 or do not reject H_0) is to be based
- A rejection region $\mathcal{R}$: the set of all test statistic values for which H_0 will be rejected

The null hypothesis will then be rejected if and only if the observed or computed test statistic value falls in the rejection region, i.e., $T \in \mathcal{R}$

- A type I error consists of rejecting the null hypothesis H_0 when it is true
- A type II error involves not rejecting H_0 when H_0 is false.
- Notations: people use α and β to denotes the probability of making Type I and Type II errors, respectively

Proposition

Suppose an experiment and a sample size are fixed and a test statistic is chosen. Then decreasing the size of the rejection region to obtain a smaller value of α results in a larger value of β for any particular parameter value consistent with H_a.

Significance level

The approach adhered to by most statistical practitioners is

- specify the largest value of α that can be tolerated
- find a rejection region having that value of α rather than anything smaller
- α: the *significance level* of the test
- the corresponding test procedure is called a *level α test*

Hypothesis testing for one parameter

1. Identify the parameter of interest
2. Determine the null value and state the null hypothesis
3. State the appropriate alternative hypothesis
4. Give the formula for the test statistic
5. State the rejection region for the selected significance level α
6. Compute statistic value from data
7. Decide whether H_0 should be rejected and state this conclusion in the problem context

Tests about a population mean

- normal population with known σ
- large-sample tests
- a normal population with unknown σ

Test about a population mean

- Null hypothesis

$$H_0 : \mu = \mu_0$$

- The alternative hypothesis will be either:
  - $H_a : \mu > \mu_0$
  - $H_a : \mu < \mu_0$
  - $H_a : \mu \neq \mu_0$

Normal population with known σ

Null hypothesis: $\mu = \mu_0$
Test statistic:

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

Alternative Hypothesis   **Rejection Region for Level α Test**

$H_a: \mu > \mu_0$ $z \geq z_\alpha$ (upper-tailed test)
$H_a: \mu < \mu_0$ $z \leq -z_\alpha$ (lower-tailed test)
$H_a: \mu \neq \mu_0$ either $z \geq z_{\alpha/2}$ or $z \leq -z_{\alpha/2}$ (two-tailed test)

z curve (probability distribution of test statistic Z when H_0 is true)

a

Shaded area
$= \alpha = P(\text{type I error})$

Rejection region: $z \geq z_\alpha$

b

Rejection region: $z \leq -z_\alpha$

c

Total shaded area
$= \alpha = P(\text{type I error})$

Shaded area
$= \alpha /2$

Shaded
area $= \alpha /2$

Rejection region: either
$z \geq z_{\alpha/2}$ or $z \leq -z_{\alpha/2}$
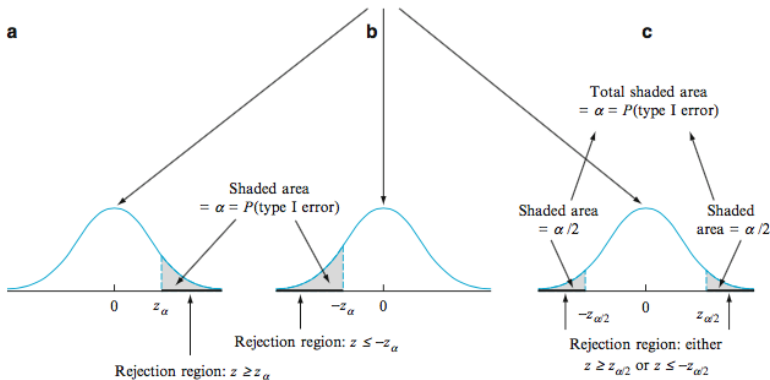
Problem

A manufacturer of sprinkler systems used for fire protection in office buildings claims that the true average system-activation temperature is $130°F$. A sample of $n = 9$ systems, when tested, yields a sample average activation temperature of $131.08°F$.

If the distribution of activation times is normal with standard deviation $1.5°F$, does the data contradict the manufacturer's claim at significance level $\alpha = 0.01$?

$\Phi(z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
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1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997

Solution

- Parameter of interest: $\mu =$ true average activation temperature
- Hypotheses

$$H_0 : \mu = 130$$
$$H_a : \mu \neq 130$$

- Test statistic:

$$z = \frac{\bar{x} - 130}{1.5/\sqrt{n}}$$

- Rejection region: either $z \leq -z_{0.005}$ or $z \geq z_{0.005} = 2.58$
- Substituting $\bar{x} = 131.08$, $n = 25 \rightarrow z = 2.16$.
- Note that $-2.58 < 2.16 < 2.58$. We fail to reject H_0 at significance level 0.01.
- The data does not give strong support to the claim that the true average differs from the design value.

Large-sample tests

Null hypothesis: $\mu = \mu_0$
Test statistic:

$$Z = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

Alternative Hypothesis	**Rejection Region for Level α Test**
H_a: $\mu > \mu_0$	$z \geq z_\alpha$ (upper-tailed test)
H_a: $\mu < \mu_0$	$z \leq -z_\alpha$ (lower-tailed test)
H_a: $\mu \neq \mu_0$	either $z \geq z_{\alpha/2}$ or $z \leq -z_{\alpha/2}$ (two-tailed test)

[Does not need the normal assumption]

Test about a normal population with unknown σ

Null hypothesis: $H_0: \mu = \mu_0$

Test statistic value: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

Alternative Hypothesis   **Rejection Region for a Level α Test**

$H_a: \mu > \mu_0$ $t \geq t_{\alpha, n-1}$ (upper-tailed)

$H_a: \mu < \mu_0$ $t \leq -t_{\alpha, n-1}$ (lower-tailed)

$H_a: \mu \neq \mu_0$ either $t \geq t_{\alpha/2, n-1}$ or $t \leq -t_{\alpha/2, n-1}$ (two-tailed)

[Require normal assumption]

Example

Problem

The amount of shaft wear (.0001 in.) after a fixed mileage was determined for each of $n = 8$ internal combustion engines having copper lead as a bearing material, resulting in $\bar{x} = 3.72$ and $s = 1.25$.

Assuming that the distribution of shaft wear is normal with mean μ, use the t-test at level 0.05 to test $H_0 : \mu = 3.5$ versus $H_a : \mu > 3.5$.

t-table

Table A.5 Critical Values for *t* Distributions

t_ν density curve

Shaded area = α

0 $t_{\alpha,\nu}$
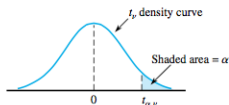
				α			
ν	.10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745

Problem

The standard thickness for silicon wafers used in a certain type of integrated circuit is 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean thickness of 246.18 μm and a sample standard deviation of 3.60 μm.

Does this data suggest that true average wafer thickness is larger than the target value? Carry out a test of significance at level .05.

P-values

Remarks

- The common approach in statistical testing is:
  1. specifying significance level α
  2. reject/not reject H_0 based on evidence
- Weaknesses of this approach:
  - it says nothing about whether the computed value of the test statistic just barely fell into the rejection region or whether it exceeded the critical value by a large amount
  - each individual may select their own significance level for their presentation
- We also want to include some *objective* quantity that describes how *strong* the rejection is $\rightarrow$ P-value

Practice problem

Problem

Suppose μ was the true average nicotine content of brand of cigarettes. We want to test:

$$H_0 : \mu = 1.5$$
$$H_a : \mu > 1.5$$

Suppose that $n = 64$ and $z = \frac{\bar{x} - 1.5}{s/\sqrt{n}} = 2.1$. Will we reject H_0 if the significance level is

(a) $\alpha = 0.05$

(b) $\alpha = 0.025$

(c) $\alpha = 0.01$

(d) $\alpha = 0.005$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
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2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
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2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997

Level of Significance α	Rejection Region	Conclusion
.05	$z \geq 1.645$	Reject H_0
.025	$z \geq 1.96$	Reject H_0
.01	$z \geq 2.33$	Do not reject H_0
.005	$z \geq 2.58$	Do not reject H_0

Question: What is the smallest value of α for which H_0 is rejected.

DEFINITION   The **P-value** (or *observed significance level*) is the smallest level of significance at which H_0 would be rejected when a specified test procedure is used on a given data set. Once the P-value has been determined, the conclusion at any particular level α results from comparing the P-value to α:

1. P-value $\leq \alpha \Rightarrow$ reject H_0 at level α.
2. P-value $> \alpha \Rightarrow$ do not reject H_0 at level α.

DECISION RULE BASED ON THE P-VALUE	Select a significance level α (as before, the desired type I error probability). Then reject H_0 if P-value $\leq \alpha$; do not reject H_0 if P-value $> \alpha$

Remark: the smaller the P-value, the more evidence there is in the sample data against the null hypothesis and for the alternative hypothesis.
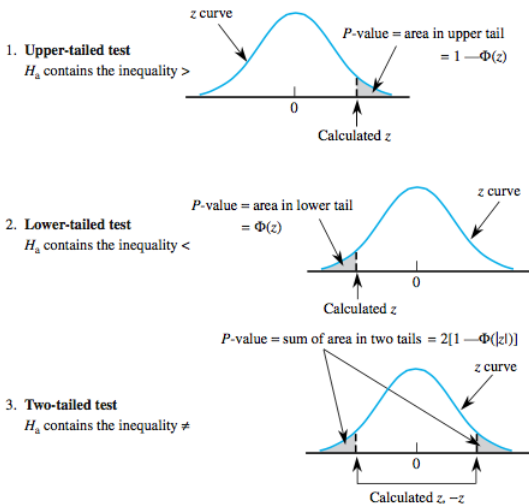
P-values for z-tests

Figure 9.7 Determination of the P-value for a z test

Practice problem

Problem

The target thickness for silicon wafers used in a certain type of integrated circuit is 245 μm. A sample of 50 wafers is obtained and the thickness of each one is determined, resulting in a sample mean thickness of 246.18 μm and a sample standard deviation of 3.60 μm.
Does this data suggest that true average wafer thickness is something other than the target value?

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
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2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997

1. **Parameter of interest:** μ = true average wafer thickness
2. **Null hypothesis:** H_0: $\mu = 245$
3. **Alternative hypothesis:** H_a: $\mu \neq 245$
4. **Formula for test statistic value:** $z = \dfrac{\bar{x} - 245}{s/\sqrt{n}}$
5. **Calculation of test statistic value:** $z = \dfrac{246.18 - 245}{3.60/\sqrt{50}} = 2.32$
6. **Determination of P-value:** Because the test is two-tailed,

$$P\text{-value} = 2[1 - \Phi(2.32)] = .0204$$

7. **Conclusion:** Using a significance level of .01, H_0 would not be rejected since .0204 > .01. At this significance level, there is insufficient evidence to conclude that true average thickness differs from the target value.

P-values for z-tests

$$P\text{-value:}\quad P = \begin{cases} 1 - \Phi(z) & \text{for an upper-tailed test} \\ \Phi(z) & \text{for a lower-tailed test} \\ 2[1 - \Phi(|z|)] & \text{for a two-tailed test} \end{cases}$$
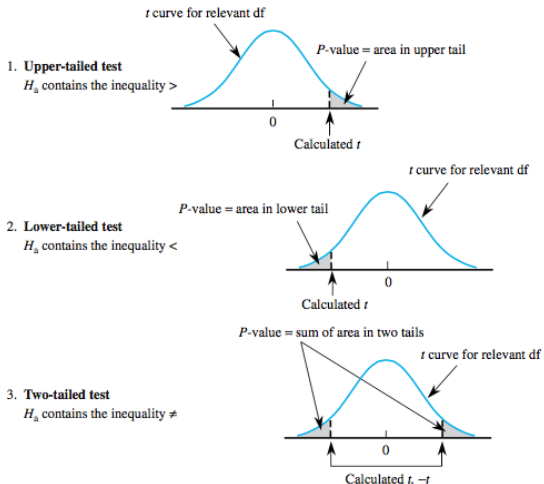
Figure 9.8 *P*-values for *t* tests

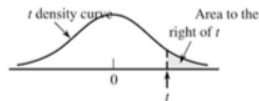## Practice problem

Problem

Suppose we want to test

$$H_0 : \mu = 25$$
$$H_a : \mu > 25$$

from a sample with $n = 5$ and the calculated value

$$t = \frac{\bar{x} - 25}{s/\sqrt{n}} = 1.02$$

(a) What is the P-value of the test

(b) Should we reject the null hypothesis?

Table A.7   *t* Curve Tail Areas

t density curve — Area to the right of *t*

t	ν 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0.0	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500
0.1	.468	.465	.463	.463	.462.	.462	.462	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461
0.2	.437	.430	.427	.426	.425	.424	.424	.423	.423	.423	.423	.422	.422	.422	.422	.422	.422	.422
0.3	.407	.396	.392	.390	.388	.387	.386	.386	.386	.385	.385	.385	.384	.384	.384	.384	.384	.384
0.4	.379	.364	.358	.355	.353	.352	.351	.350	.349	.349	.348	.348	.348	.347	.347	.347	.347	.347
0.5	.352	.333	.326	.322	.319	.317	.316	.315	.315	.314	.313	.313	.313	.312	.312	.312	.312	.312
0.6	.328	.305	.295	.290	.287	.285	.284	.283	.282	.281	.280	.280	.279	.279	.279	.278	.278	.278
0.7	.306	.278	.267	.261	.258	.255	.253	.252	.251	.250	.249	.249	.248	.247	.247	.247	.247	.246
0.8	.285	.254	.241	.234	.230	.227	.225	.223	.222	.221	.220	.220	.219	.218	.218	.218	.217	.217
0.9	.267	.232	.217	.210	.205	.201	.199	.197	.196	.195	.194	.193	.192	.191	.191	.191	.190	.190
1.0	.250	.211	.196	.187	.182	.178	.175	.173	.172	.170	.169	.169	.168	.167	.167	.166	.166	.165
1.1	.235	.193	.176	.167	.162	.157	.154	.152	.150	.149	.147	.146	.146	.144	.144	.144	.143	.143
1.2	.221	.177	.158	.148	.142	.138	.135	.132	.130	.129	.128	.127	.126	.124	.124	.124	.123	.123
1.3	.209	.162	.142	.132	.125	.121	.117	.115	.113	.111	.110	.109	.108	.107	.107	.106	.105	.105
1.4	.197	.148	.128	.117	.110	.106	.102	.100	.098	.096	.095	.093	.092	.091	.091	.090	.090	.089
1.5	.187	.136	.115	.104	.097	.092	.089	.086	.084	.082	.081	.080	.079	.077	.077	.077	.076	.075

A P-value:

- is not the probability that H_0 is true
- is not the probability of rejecting H_0
- is the probability, calculated assuming that H_0 is true, of obtaining a test statistic value at least as contradictory to the null hypothesis as the value that actually resulted