MATH 637 – Homework 0

Due: 2/21/2022 11:59PM

This homework serves as a self-assessment whether you have enough back-ground knowledge for the course. It will not be graded, but an attempt of all questions (regardless of the correctness) will earn you 2% bonus toward the final grade.

Submit your solutions to Canvas as a PDF file. You may scan a handwritten document, but they will be returned ungraded if they are not legible.

Question 1 (Linear Algebra)

Find the largest eigenvalue of:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 5 \\ 0 & 4 & 3 \end{bmatrix}$$

and one of its corresponding eigenvectors.

Question 2 (Probability)

- Consider a sample $x_1, x_2, \ldots, x_n$ of size n from a population, where x_i's are real numbers. Write the formulas for the sample mean and sample variance.
- When a batch of a certain chemical product is prepared, the amount of a particular impurity in the batch is a random variable with mean value 4.0 g and standard deviation 1.5 g. If 50 batches are independently prepared, what is the (approximate) probability that the sample average amount of impurity is between 3.5 and 3.8 g?

Question 3 (Calculus)

Consider three points $A = (1, 0)$, $B = (0, 1)$, $C = (0, 0)$ on the plane $\mathbb{R}^2$ and let $\mathcal{L}$ be the line described by the equation

$$ax + by + c = 0$$

where $a, b, c \in \mathbb{R}$ and $a^2 + b^2 = 1$.

Denote by $d(P, \mathcal{L})$ the distance from a point P to the line $\mathcal{L}$. Find the values of a, b, c such that:

$$d(A, \mathcal{L})^2 + d(B, \mathcal{L})^2 + d(C, \mathcal{L})^2$$

is minimized.

Question 4 (Programming)

- Write pseudocode (or code in any programming language) of a function that takes in 2 numbers a and b and returns the larger value. You must use if ... else ... in your function.
- Write pseudocode (or code in any programming language) to compute the sample mean and sample variance in Question 2.