MATH 637 – Homework 4

Due: Monday, May 2nd, 11:59PM

Submit your solutions to Canvas as a PDF file. You may scan (or take a good picture of) a handwritten document, but they will be returned ungraded if they are not legible.

You can use any results that have been stated/proven in class.

Question 1

Let X be a non-negative random variable with $E[X^4] = 1$. Show that

$$P[X \geq 2] < 0.5$$

Question 2

Let $X_1, X_2, \ldots, X_{100}$ be i.i.d copies of a random variable $X \in [-1, 1]$ and $E[X] = 0$. Denote

$$S = X_1 + X_2 + \ldots + X_{100}$$

Show that

$$P\left[S \geq 50\right] \leq 0.05$$

Question 3

Suppose that the data set $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$ are generated by the following rules:

- $x_i = (x_i^{(1)}), x_i^{(2)})$ are chosen uniformly at random on the domain $[-3, 3] \times [-3, 3]$
- the label y_i is computed by

$$y_i = \text{sign}\left((x_i^{(2)})^2 - (x_i^{(1)})^3 + 2(x_i^{(1)}) - 1\right)$$

Construct a map $\phi : \mathbb{R}^2 \to \mathbb{R}^p$ for some $p > 2$ such that the mapped dataset $\{(x_1', y_1), (x_2', y_2), \ldots, (x_n', y_n)\}$ where

$$x_i' = \phi(x_i)$$

is linearly separable (i.e., separable by a hyperplane in $\mathbb{R}^p$).