Mathematical techniques in data science

Lecture 11: Deep learning with Keras

Feed-forward neural networks

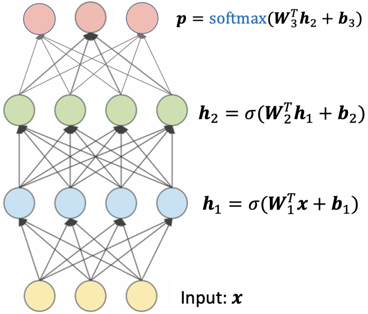
- Structure:
  - Graphical representation
  - Activation functions
  - Loss functions
- Training:
  - Stochastic gradient descent
  - Back-propagation

Settings

- Data:
  $(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_n, y_n)$
- Model parameters:

  $$\theta = (W_1, b_1, W_2, b_2, \ldots, W_L, b_L)$$

- Training: Find the best value of θ that fits the data



One-hot encoding

id	color
1	red
2	blue
3	green
4	blue

One Hot Encoding →

id	color_red	color_blue	color_green
1	1	0	0
2	0	1	0
3	0	0	1
4	0	1	0

Convert a categorical value into a binary vector with exactly one "1" element, and the rest are 0

Stochastic gradient descent: terminology

- Mini-batch stochastic gradient descent
  - randomly shuffle examples in the training set, divide them into k mini-batches of data of size m
  - for each batch I_i (i=1, . . . , k), approximate the objective function by

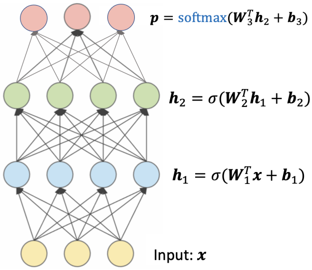
$$\hat{\ell}(\theta) = \frac{1}{m} \sum_{j \in I_i} L(\theta, x_j, y_j)$$

    and update θ

$$\theta \leftarrow \theta - \rho \nabla \hat{\ell}(\theta)$$

  - Repeat until an approximate minimum is obtained or a maximum numbers M epochs are done
- Terminology:
  - m: batch-size
  - ρ: learning rate
  - M: number of epochs

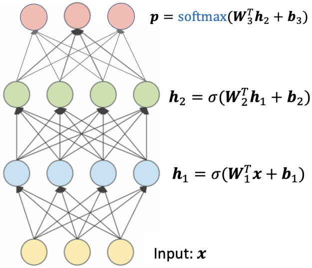
Back-propagation



Use chain rule to compute $\nabla \ell(\theta)$

$$\frac{\partial \ell}{\partial b_1} = \frac{\partial \ell}{\partial p}(p) \cdot \frac{\partial p}{\partial h_2}(h_2, W_3, b_3) \cdot \frac{\partial h_2}{\partial h_1}(h_1, W_1, b_1) \cdot \frac{\partial h_1}{\partial b_1}(x, W_1, b_1)$$

Back-propagation



- One forward pass to evaluate h_1, h_2, p, ℓ
- One backward pass to compute $\nabla \ell(\theta)$

Back-propagation

- Advantage: The cost to compute the partial derivatives with respect to all parameters are just twice the cost of a forward evaluations
- Drawback: The functions used to describe the network (activation functions and loss functions) needs to belong to the class of functions supported by the computational platform

Feed-forward neural networks

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sklearn.neural_network.MLPClassifier

class sklearn.neural_network.MLPClassifier(*hidden_layer_sizes=(100,), activation='relu', *, solver='adam', alpha=0.0001, batch_size='auto', learning_rate='constant', learning_rate_init=0.001, power_t=0.5, max_iter=200, shuffle=True, random_state=None, tol=0.0001, verbose=False, warm_start=False, momentum=0.9, nesterovs_momentum=True, early_stopping=False, validation_fraction=0.1, beta_1=0.9, beta_2=0.999, epsilon=1e-08, n_iter_no_change=10, max_fun=15000*) [source]

Multi-layer Perceptron classifier.

This model optimizes the log-loss function using LBFGS or stochastic gradient descent.

New in version 0.18.

Parameters:

hidden_layer_sizes : *tuple, length = n_layers - 2, default=(100,)*

The ith element represents the number of neurons in the ith hidden layer.

activation : *{'identity', 'logistic', 'tanh', 'relu'}, default='relu'*

Activation function for the hidden layer.

- 'identity', no-op activation, useful to implement linear bottleneck, returns f(x) = x
- 'logistic', the logistic sigmoid function, returns f(x) = 1 / (1 + exp(-x)).
- 'tanh', the hyperbolic tan function, returns f(x) = tanh(x).
- 'relu', the rectified linear unit function, returns f(x) = max(0, x)

Keras

Simple. Flexible. Powerful.

- High level API for deep learning
- More flexible to define network architecture than sklearn

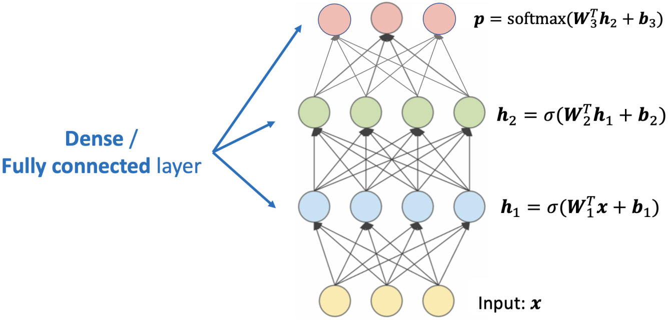
Define network architecture (1)

- Define a network as a Sequential object
- Add layers to it one-by-one

```
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense

model = Sequential()
model.add(Dense(50, activation='relu'))
model.add(Dense(50, activation='relu'))
model.add(Dense(10, activation='softmax'))
```

Define network architecture (2)



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Labels in Keras are usually encoded as one-hot vectors

Demo: train an MLP using Keras

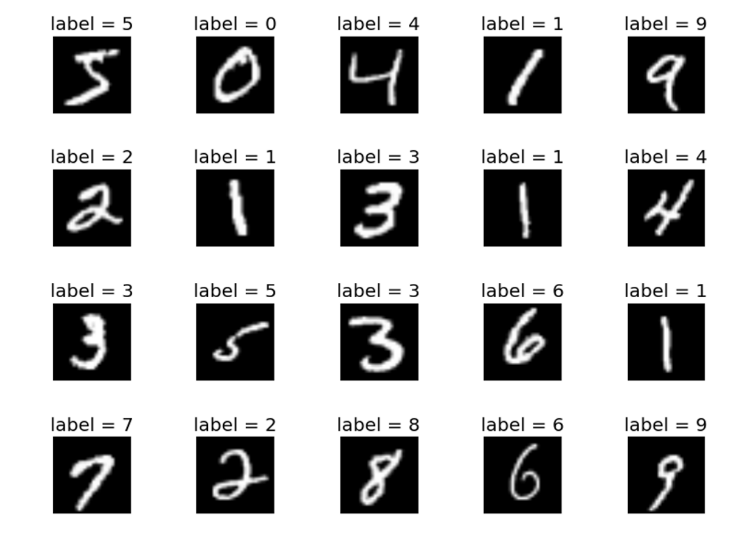
Some intros to computer vision

Computer vision

A field that enables computers and systems to derive meaningful information from digital images, videos and other visual inputs

- Image classification/object recognition
- Object detection
- Image segmentation
- Image generation
- Image style transfer

Image classification



Object detection

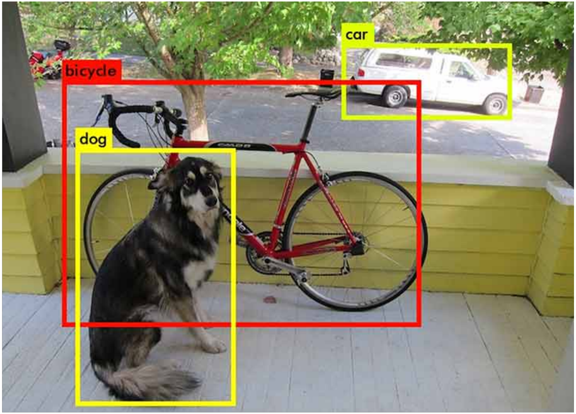


Image segmentation

Image generation

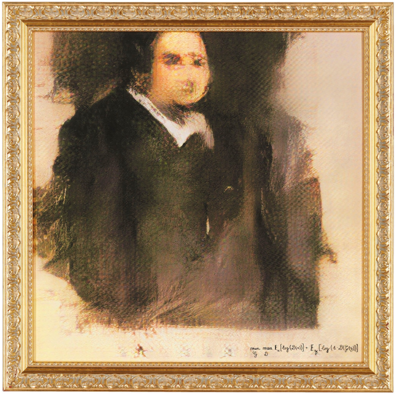
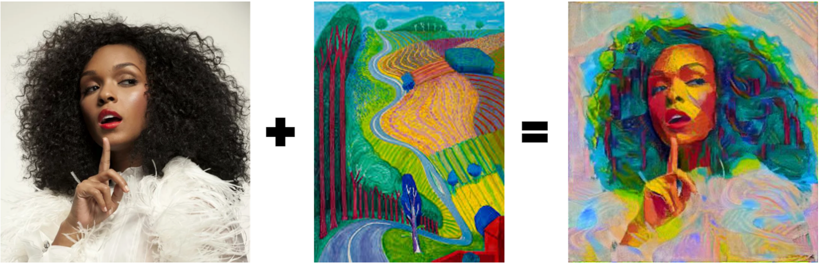
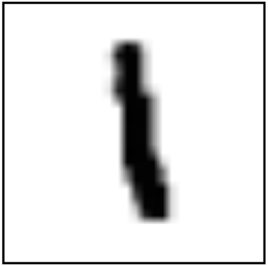


Image style transfer

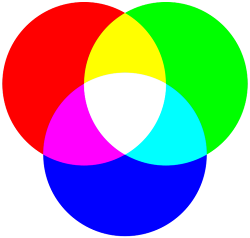


Grayscale image representation



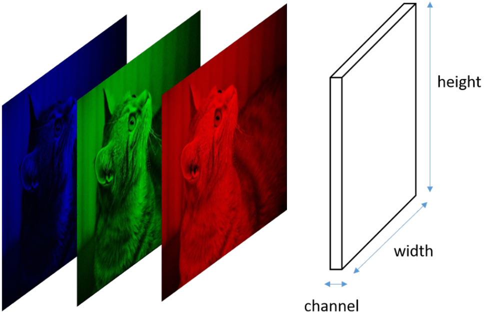
$$\cong \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&.6&.8&0&0&0&0&0&0\\
0&0&0&0&0&0&.7&1&0&0&0&0&0&0\\
0&0&0&0&0&0&.7&1&0&0&0&0&0&0\\
0&0&0&0&0&0&.5&1&.4&0&0&0&0&0\\
0&0&0&0&0&0&0&1&.4&0&0&0&0&0\\
0&0&0&0&0&0&0&1&.4&0&0&0&0&0\\
0&0&0&0&0&0&0&1&.7&0&0&0&0&0\\
0&0&0&0&0&0&0&1&1&0&0&0&0&0\\
0&0&0&0&0&0&0&.9&1&.1&0&0&0&0\\
0&0&0&0&0&0&0&.3&1&.1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}$$

Color image representation



- Use RGB color mode
- Represent a color by 3 values: R (Red) G (Green) B (Blue)
- There are other color modes

Image representation



- An image is an H x W x C matrix: H (height), W (width), C (depth or number of channels)
- Grayscale image: C = 1
- RGB image: C = 3