MATH 637 – Homework 4

Due: April 25th, 5:15 PM

Submit your solutions to Canvas as a PDF file. You may scan (or take a good picture of) a handwritten document, but they will be returned ungraded if they are not legible.

You can use any results that have been stated/proven in class.

Question 1 (3%)

Consider the following independent random variables

- X: a Bernoulli random variable with probability of success 0.9. That is:

$$X = \begin{cases} 1 & \text{with probability } 0.9 \\ 0 & \text{with probability } 0.1 \end{cases}$$

- $Y_1, Y_2, \ldots Y_{25}$ are independent and

$$Y_i = \begin{cases} 2 & \text{with probability } 0.05 \\ 0 & \text{with probability } 0.95 \end{cases} \quad \text{for } 1 \le i \le 25$$

(a) Compute $P[X > Y_i]$

(b) Compute

$$P\left[X > \max_{1 \le i \le 25} \{Y_i\}\right]$$

Question 2 (2%)

Let X be a non-negative random variable with $E[\ln(X)] = 2$. Show that

$$P[X \ge 1000] < 0.3$$

Question 3 (3%)

Let $X_1, X_2, \ldots, X_{100}$ be i.i.d copies of a random variable $X \in [-1, 1]$ and $E[X] = 0$. Denote

$$S = X_1 + X_2 + \ldots + X_{100}$$

Show that

$$P\left[|S| \ge 60\right] < 0.05$$

Question 4 (2%)

Given a class $\mathcal{H}$ of real-valued functions that map the input space $\mathcal{X}$ to $\mathcal{Y} = \{-1, 1\}$ and a sample $S = \{x_1, x_2, \ldots, x_n\} \in \mathcal{X}$, the empirical Rademacher complexity of $\mathcal{H}$ given S is defined as:

$$Rad(\mathcal{H}) = \mathbb{E}_{\sigma_1, \sigma_2, \ldots, \sigma_m} \left[\sup_{f \in \mathcal{H}} \frac{1}{m} \sum_{i=1}^{m} \sigma_i f(x_i) \right]$$

where $\sigma_1, \sigma_2, \ldots, \sigma_m$ are independent random variables drawn from the Rademacher distribution

$$P[\sigma_i = 1] = P[\sigma_i = -1] = 1/2.$$

Show that $0 \leq Rad(\mathcal{H}) \leq 1$.