Mathematical techniques in data science

Lecture 6: Deep learning with Keras

Feed-forward neural networks

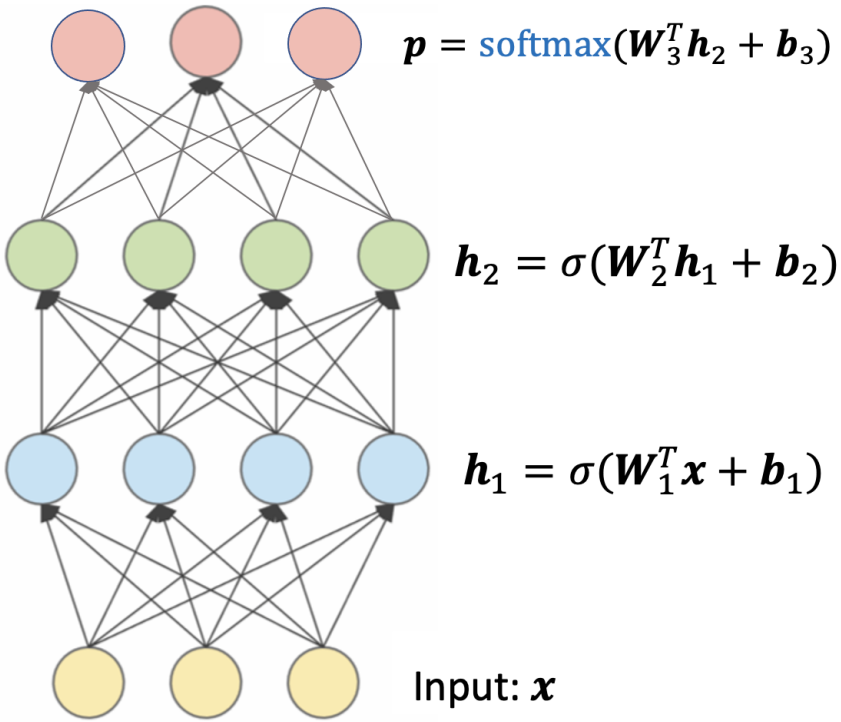
- Structure:
  - Graphical representation
  - Activation functions
  - Loss functions
- Training:
  - Stochastic gradient descent
  - Back-propagation

Settings

- Data:
  $(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_n, y_n)$
- Model parameters:

  $\theta = (W_1, b_1, W_2, b_2, \ldots, W_L, b_L)$

- Training: Find the best value of θ that fits the data



Stochastic gradient descent: terminology

- Mini-batch stochastic gradient descent
  - randomly shuffle examples in the training set, divide them into k mini-batches of data of size m
  - for each batch I_i (i=1, . . . , k), approximate the objective function by

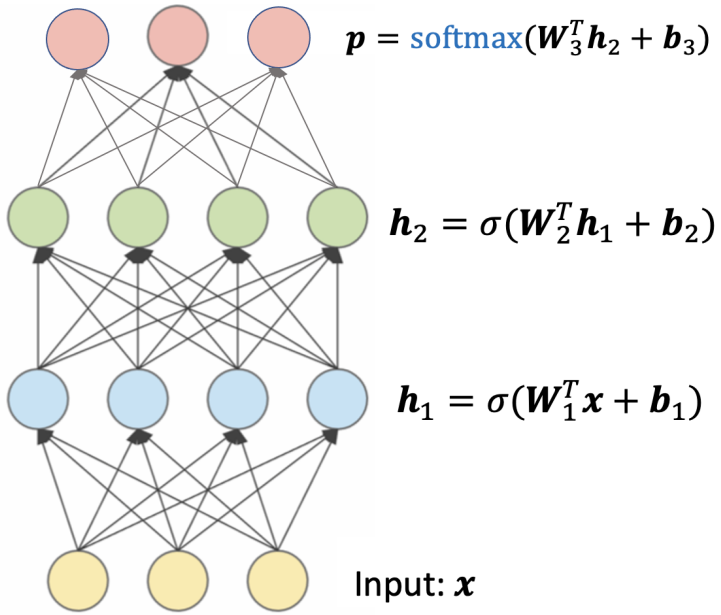
$$\hat{\ell}(\theta) = \frac{1}{m} \sum_{j \in I_i} L(\theta, x_j, y_j)$$

    and update θ

$$\theta \leftarrow \theta - \rho \nabla \hat{\ell}(\theta)$$

  - Repeat until an approximate minimum is obtained or a maximum numbers M epochs are done
- Terminology:
  - m: batch-size
  - ρ: learning rate
  - M: number of epochs

Back-propagation



- One forward pass to evaluate h_1, h_2, p, ℓ
- One backward pass to compute $\nabla \ell(\theta)$

sklearn.neural_network.MLPClassifier

class sklearn.neural_network.MLPClassifier(*hidden_layer_sizes=(100,)*, *activation='relu'*, ***, *solver='adam'*, *alpha=0.0001*, *batch_size='auto'*, *learning_rate='constant'*, *learning_rate_init=0.001*, *power_t=0.5*, *max_iter=200*, *shuffle=True*, *random_state=None*, *tol=0.0001*, *verbose=False*, *warm_start=False*, *momentum=0.9*, *nesterovs_momentum=True*, *early_stopping=False*, *validation_fraction=0.1*, *beta_1=0.9*, *beta_2=0.999*, *epsilon=1e-08*, *n_iter_no_change=10*, *max_fun=15000*) [source]

Multi-layer Perceptron classifier.

This model optimizes the log-loss function using LBFGS or stochastic gradient descent.

New in version 0.18.

Parameters:

hidden_layer_sizes : *tuple, length = n_layers - 2, default=(100,)*
The ith element represents the number of neurons in the ith hidden layer.

activation : *{'identity', 'logistic', 'tanh', 'relu'}, default='relu'*
Activation function for the hidden layer.

- 'identity', no-op activation, useful to implement linear bottleneck, returns f(x) = x
- 'logistic', the logistic sigmoid function, returns f(x) = 1 / (1 + exp(-x)).
- 'tanh', the hyperbolic tan function, returns f(x) = tanh(x).
- 'relu', the rectified linear unit function, returns f(x) = max(0, x)

- High level API for deep learning
- More flexible to define network architecture than sklearn

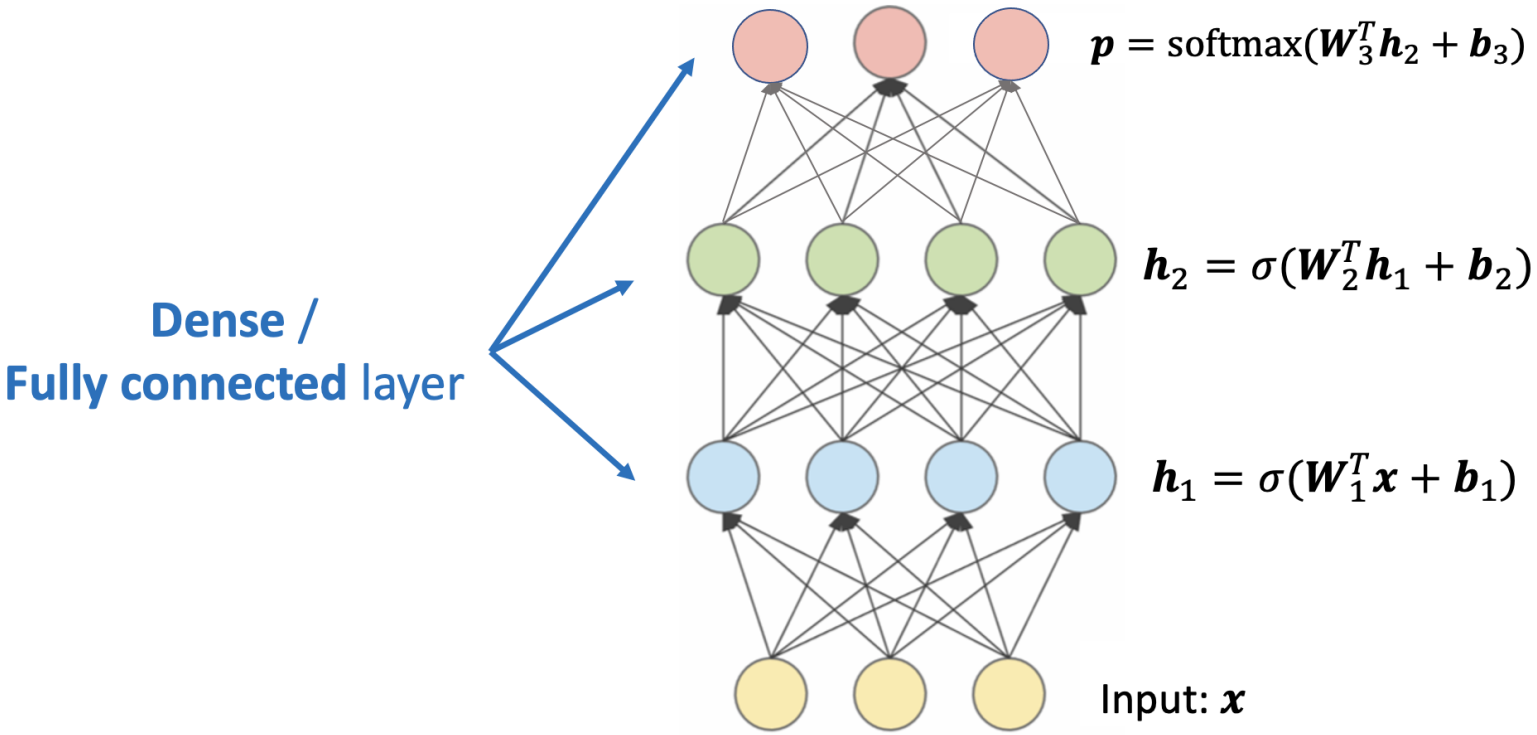
Define network architecture (1)

- Define a network as a Sequential object
- Add layers to it one-by-one

```
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense

model = Sequential()
model.add(Dense(50, activation='relu'))
model.add(Dense(50, activation='relu'))
model.add(Dense(10, activation='softmax'))
```

Define network architecture (2)



One-hot encoding

id	color
1	red
2	blue
3	green
4	blue

id	color_red	color_blue	color_green
1	1	0	0
2	0	1	0
3	0	0	1
4	0	1	0

Labels in Keras are usually encoded as one-hot vectors

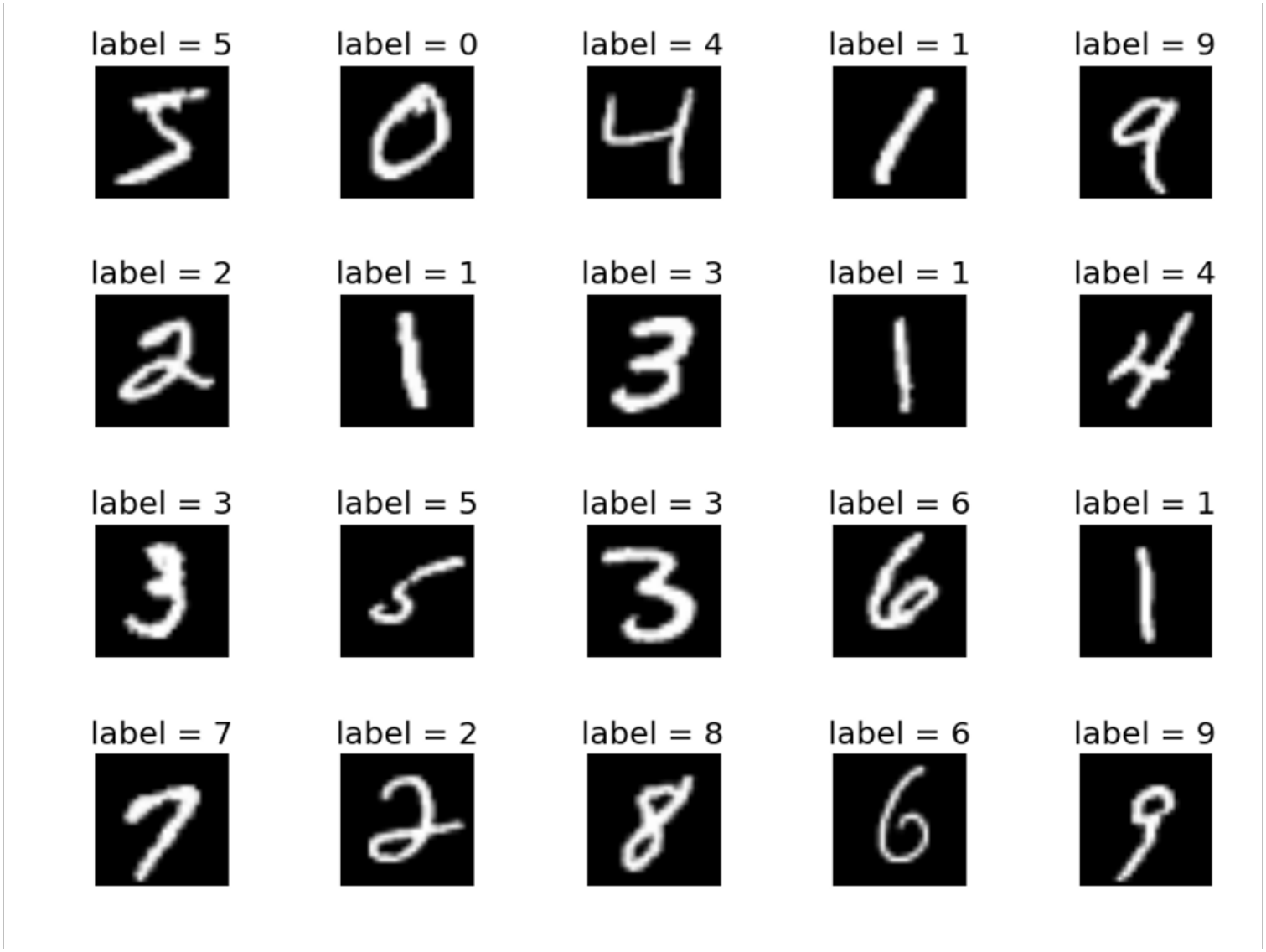
Some intros to computer vision

Computer vision

A field that enables computers and systems to derive meaningful information from digital images, videos and other visual inputs

- Image classification/object recognition
- Object detection
- Image segmentation
- Image generation

Image classification



Object detection

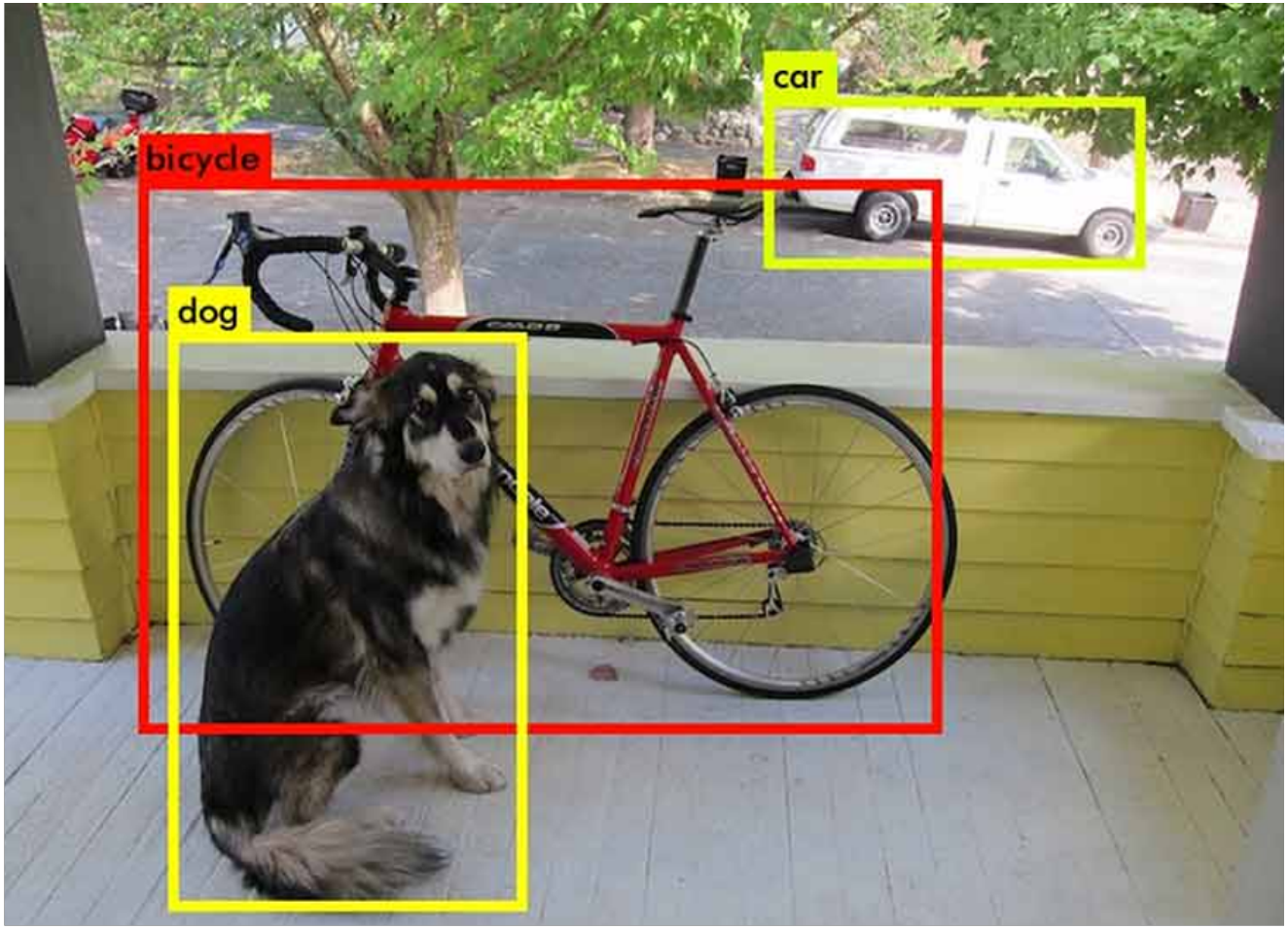
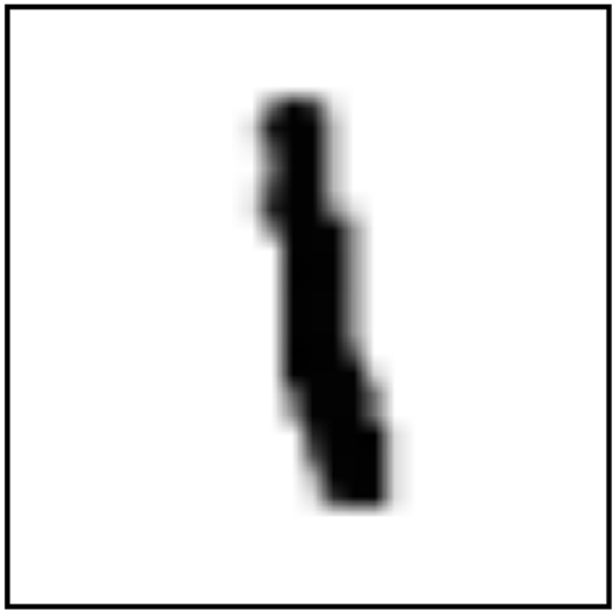


Image segmentation

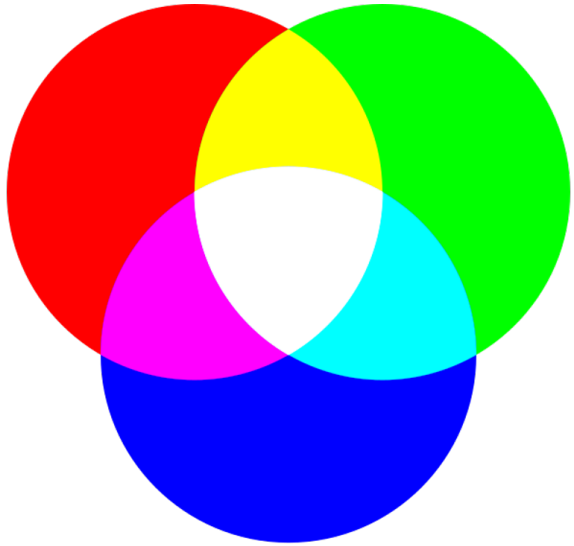
Grayscale image representation



$\approx$

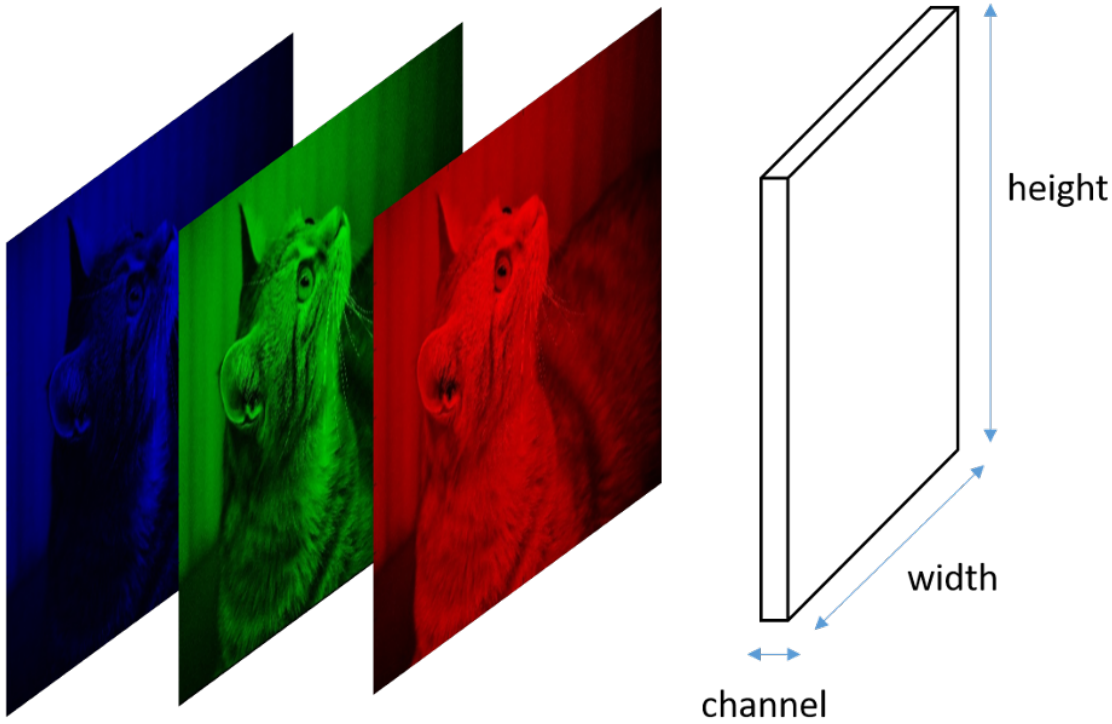
$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .6 & .8 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .7 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .7 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .5 & 1 & .4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & .4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & .4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & .7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & .9 & 1 & .1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & .3 & 1 & .1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Color image representation



- Use RGB color mode
- Represent a color by 3 values: R (Red) G (Green) B (Blue)
- There are other color modes

Image representation



- An image is an H x W x C matrix: H (height), W (width), C (depth or number of channels)
- Grayscale image: $C = 1$
- RGB image: $C = 3$

Demo: train an MLP using Keras